MATHEMATICS

Grade 5

Student Textbook

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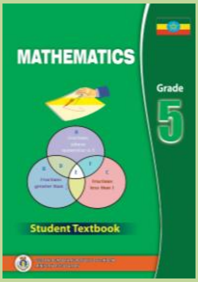

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UNIT ONE

WHOLE NUMBERS AND THE FOUR OPERATIONS

Unit Outcomes: After completing this unit, you should be able to:

- understand and have deep knowledge about whole numbers.
- perform the four fundamental operations on whole numbers.
- apply your knowledge of whole numbers to solve problems in your environment.

Introduction

In earlier grades, you have learnt about whole numbers up to 1,000,000, their properties and basic mathematical operations upon them. After a review of your knowledge about whole numbers, you will continue studying whole numbers greater than 1,000,000, and the four operations in the present unit.

1.1 Whole Numbers Greater Than 1,000,000

1.1.1 Revision of Whole Numbers Up to 1,000,000

Activity 1.1

1. What is the name given to the numbers 0, 1, 2, 3, 4, 5,... ?
2. What is the least whole number? Is there any largest whole number?
3. Identify a possible pattern. Use the pattern to write the next four numbers.

a) 10,000, 20,000, 30,000, _____, _____, ______, _____

b) 100,000, 200,000, 300,000, ____, ____ , ___ , _____

Do you remember how to read and write whole numbers up to 1,000,000? In your previous study of Mathematics lessons on whole numbers, you have learnt about place value.

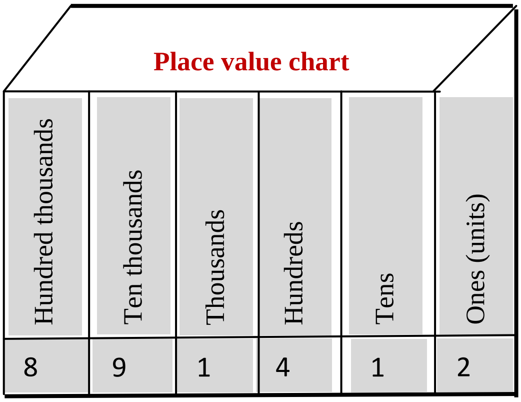


Figure 1.1

How do you read the whole number 891,412? Can you write the whole number 'three hundred seventeen thousand sixty' and show to your partner? In order to help you revise the lessons on whole numbers up to 1,000,000 you have studied earlier attempt each of the problems given in the following exercise.

Exercise 1A

1. Read these numbers.

   a. 136,042
   b. 218,606
   c. 390,071
   d. 467,319
   e. 522,202
   f. 650,505
   g. 800,304
   h. 430,713

2. Match a number with its word expression.

Column A	**Column B**
i. 100,003	a. five hundred forty thousand eight hundred nine
ii. 430,006	b. One hundred thousand three
iii. 896,750	c. Four hundred thirty thousand six
iv. 540,809	d. Three hundred eighteen thousand fourteen
v. 318,014	e. Eight hundred ninety six thousand seven hundred fifty
vi. 594,713	f. Three hundred seventeen thousand sixty five
vii. 405,028	g. Four hundred five thousand twenty eight
viii. 317,065	h. Five hundred ninety four thousand seven hundred thirteen
	i. One hundred thousand thirty
	j. Four hundred five thousand eighty two
	k. Three hundred eighteen thousand fourty

3. Write these numbers in words.

   a. 100,350
   b. 206,570
   c. 160,080
   d. 320,010
   e. 485,675
   f. 860,003
   g. 973,468
   h. 98,764

4. Write down the place value of 6 in each of these numbers.

   a. 324,761 b. 406,117 c. 218,416 d. 163,514 e. 258,629

5. Write these numbers in figures (the first one is done for you)

   a. One hundred forty thousand 140,000.
   b. One hundred seventy thousand six hundred thirty.
   c. Two hundred five thousand three hundred eighty.
   d. Five hundred sixteen thousand four hundred nine.

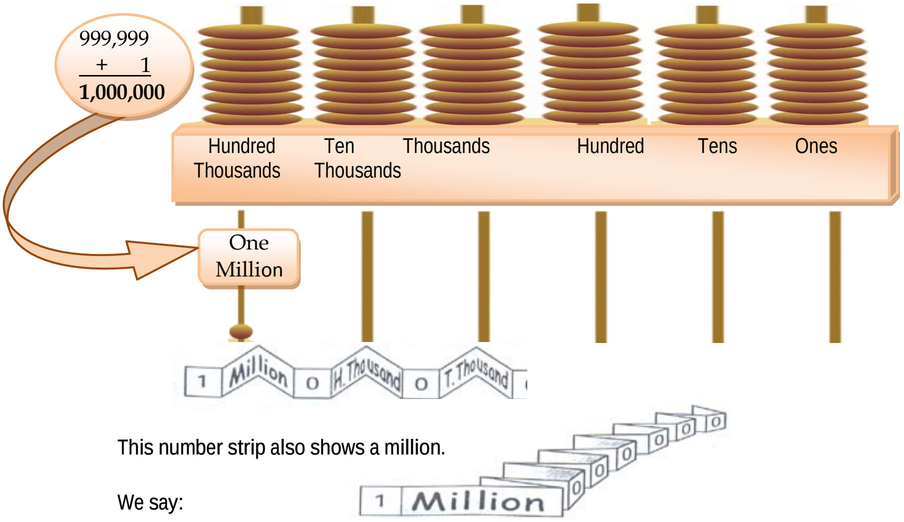
e. Six hundred three thousand twenty seven.
f. Ninety thousand seventy four.
g. Seven hundred eighty five thousand two hundred twelve.

6. Comparing and ordering: Draw a line under the greatest number in each group.

a. 97,000	b. 388,000	c. 689,400
705,000	326,000	652,800
423,000	362,000	630,900

1.1.2 Whole Numbers Greater Than 1,000,000

What number comes after 999,999?



This number strip also shows a million.

We say:

Figure 1.2

We need a new place value. Ten hundred thousand make a thousand thousands which is a **million**. You can see that one million is a 7 digit number. When do we count in millions?

Have you heard people talking about millions? What do you know about stars and planets?

Did you know that the sun is about 150,000,000 km away from Earth?

Find out the names of the other planets and how far they are from the sun, where 1 Mile ≈ 1.6km.

Figure 1.3

Planet	*Miles from the sun*	*Km from the sun*	*Planet*	*Miles from the sun*	*Km from the sun*
Mercury	**36 million**		**Saturn**	**886.1 million**	
Venus	**67.2 million**		**Uranus**	**1783 million**	
Earth	**92.9 million**		**Neptune**	**2793 million**	
Mars	**141.5 million**		**Pluto**	**3670 million**	
Jupiter	**483.3 million**				

Do you know (according to CSA, 2007) that the population of Ethiopia is about 74 million?

There are more people in India than in most countries. India alone has more than 900 Million people. How many times is India's population bigger than Ethiopia's population? What is the population of your region?

Activity 1.2

Write these numbers in words. The first one is done for you.

a. 3,500,820 three million five hundred thousand eight hundred twenty

b. 7,416,035 ________________________________

c. 8,042,107 ________________________________

d. 9,104,060 ________________________________

e. 12,000,000________________________________

Study the following example

Example 1

Writing numbers in figures. Say this number: Three Million four hundred seventy thousand fifty.

How will you write this in figures?

Remember we break up the numbers and write them in groups, like this.

Three million		**3,000,000**
Four hundred seventy thousand	**+**	**470,000**
Fifty		**50**
		3,470,050

Group work 1.1

1. Convert 8000 kilometers in to meters.
2. Convert 900 kilometers in to centimeters.

In addition to reading and writing whole numbers, you can also find the predecessor (except zero) and successor of a whole number.

Example 2

a) **What whole number comes before 3,465,287? The number that comes before 3,465,287 is less by 1. That is, 3,465,287-1. Therefore 3,465,286 is the Predecessor of 3,465,287.**

b) **What whole number comes after 2,746,352?**
Remember that the number that comes after 2,746,352 is greater by 1. That is 2,746,352 + 1. Therefore, 2,746,353 is the successor of 2,746,352.

Note

1. **Any whole number n different from 0 has a predecessor "n-1" and a successor "n+1".**
2. **There is no largest whole number (why?)**
3. **Zero is the smallest whole number.**

Exercise 1B

1. Write these numbers in figures.
   a. Five million, eight hundred four thousand, twenty.
   b. Eight million, nine hundred six thousand, one hundred thirty two.
   c. Nine million thirty thousand, four hundred three.
2. Write the numbers in words. The first one is done for you.
   a. A heart beats about 37,000,000 times each year.

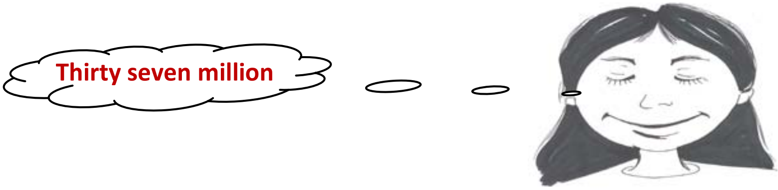


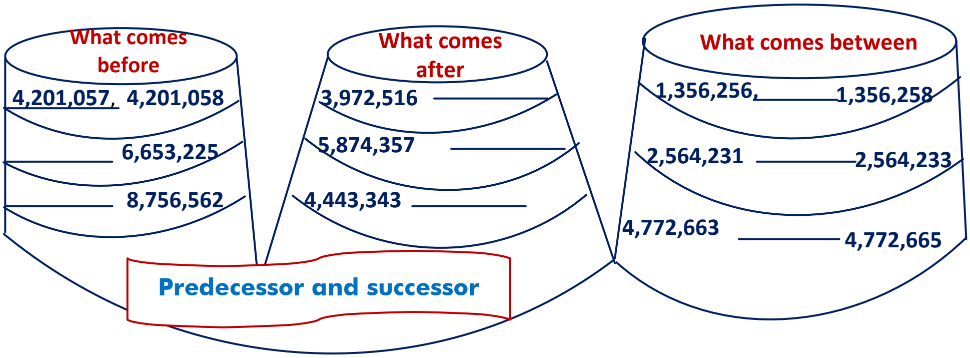
Figure 1.4

   b. Most people blink about 5,625,000 times each year ____________
   c. One Megabyte is 1,048,576 bytes.______________
   d. Africa has an area of 30,271,000km^2 ____________

3. Determine the predecessor and successor of each the following numbers.

	Predecessor	Successor
a. 3,406,705	__________________	__________________
b. 5,167,428	__________________	__________________
c. 9,582,396	__________________	__________________
d. 8,005,104	__________________	__________________
e. 6,767,221	__________________	__________________

4. Write each missing number.



5. Compare the numbers using >, < or =. The first one is done for you.

a. 5,370,002 $\boxed{>}$ 5,370,001

b. 3,820,013 ☐ 3,820,012

c. 6,540,000 ☐ 540,000 + 6,000,000

d. 7,630,009 ☐ 7,630,010

e. 8,999,026 ☐ 8,999,025

1.1.3 Place Value and Ordering of Whole Numbers

Activity 1.3

Arrange the digits of the number 214,587 to get the

a) *smallest six digit number possible*

b) *largest six digit number possible*

Remember that you have learnt about place value and ordering of whole numbers upto 1,000,000. Here you will learn about place value and ordering of whole numbers in more detail.

a) Finding the place value of a digit in a whole number

The position of each digit in a number determines its **place value**. A place use the same chart is shown next for the whole number 48,337,000.

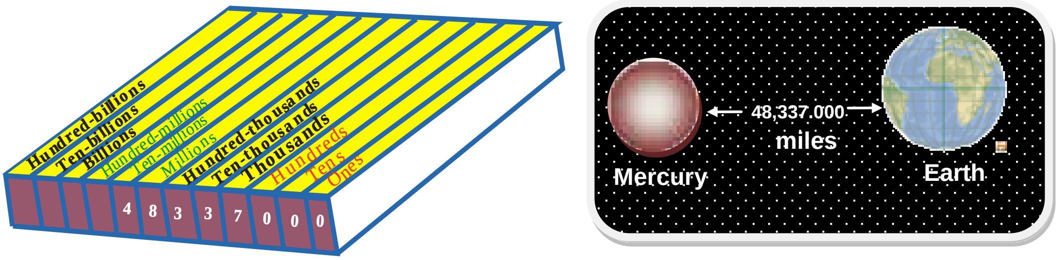


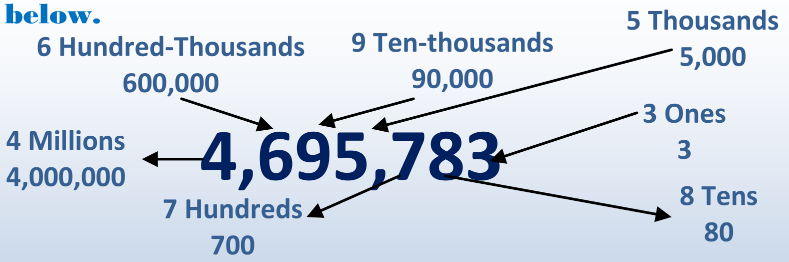
Figure 1.5

The two 3s in 48,337,000 represent different amounts because of their different placements. The place value of the 3 on the left is hundred-thousands. The place value of the 3 on the right is ten-thousands.

Study the following examples

Example 3

The place value of the digits of the number 4,695,783 is shown below.



We write the expansion of the given number as

$$4{,}695{,}783 = 4{,}000{,}000 + 600{,}000 + 90{,}000 + 5{,}000 + 700 + 80 + 3$$
$$= (4\times1{,}000{,}000)+(6\times100{,}000)+(9\times10{,}000)$$
$$+(5\times1{,}000)+(7\times100)+(8\times10)+3$$

Example 4

The palce value of the digits of the number 5,793,612 is shown below

Place Value chart

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1,000,000	**100,000**	**10,000**	**1,000**	**100**	**10**	**1**
5	**7**	**9**	**3**	**6**	**1**	**2**

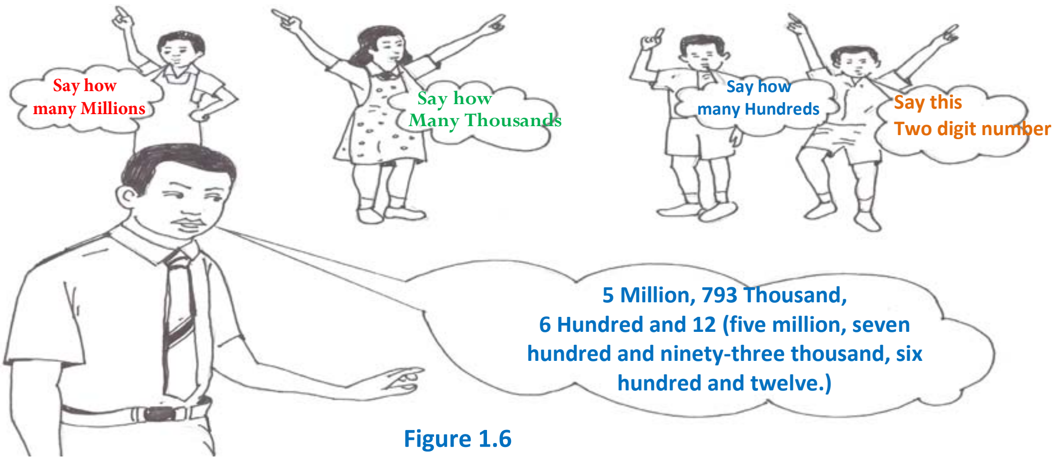


Figure 1.6

The expansion is as follows:

$$5{,}793{,}612 = (5\times1{,}000{,}000) + (7\times100{,}000) + (9\times10{,}000) + (3\times1{,}000) + (6\times100) + (1\times10) + 2$$

Group work 1.2

1. What is the place value of 7 in the whole number 27,431,568?
2. Write the expansion of the whole number 8,697,351.

b) Ordering of whole numbers

In addition to telling the place value of whole numbers you can also order and compare them.

Study the following examples

Example 5

We are going to compare large numbers and order them.

a. **Putting numbers in order:**
   To put numbers in order, look at the digits with the same place value. Start at the left.
   7546 > 7364

Thousands	Hundreds	Tens	Units
7	5	4	6
7	3	6	4

Look here next

Look here first

The thousands digits are the same.
The hundreds digits are 5 and 3.
5 > 3
So 7546 > 7364

b. Compare and order

712,340 529,798 645,938 1,306,493 6,790,104

645,349 5,438,654 2,009,870 917,503 4,877,428 689,740

First arrange them vertically with the ones in a line.

712,340
529,798
645,938
1,306,493
6,790,104
645,349
5,438,654
2,009,870
917,503
4,877,428
689,740

Step 1. List the numbers with the largest number of digits

Step 2. Compare the highest place value digits and order.

Step 3. When there are equal digits re-order using the next lower digit

Step 4. Order other groups of numbers with equal number of digits using steps 1,2 and 3

6,790,104
5,438,654
4,877,428
2,009,870
1,306,493
917,503
712,340
689,740
645,938
645,349
529,798

Exercise 1.C

1. Write the place value of the underlined number.

a. $\underline{7}$,816,489 c. 4,$\underline{9}$76,096 e. 2,6$\underline{4}$8,143

b. 6,594,$\underline{0}$38 d. 3,80$\underline{0}$,667

2. Write the following numbers in expanded form.

a. 2,536,879 c. 7,089,461 e. 9,988,472

b. 1,546,308 d. 8,571,026

3. Write the number for the following.

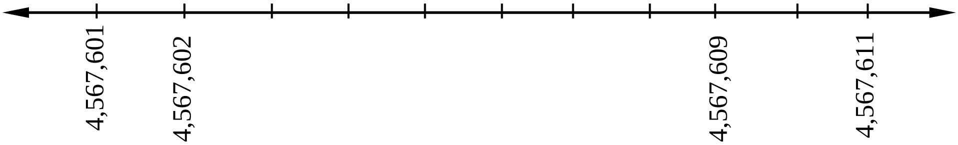
a. (3 × 1,000,000) + (6 × 10,000) + (8 × 100) + (4 × 10) + 3

b. (6 × 1,000,000) + (8 × 100,000) + (7 × 1,000) + (3 × 10) + 9

c. (4 × 1,000,000) + (5 × 1,000) + (6 × 100) + 7

d. (8 × 1,000,000) + (3 × 100) + 8

4. Which numbers are missing?



5. Write a number which has:

a. 6 digits, with 8 in the Ten Thousands position.

b. 7 digits, with 7 in the Hundred Thousands position.

c. 7 digits, with 3 in the millions position.

6. Compare the following using >, < or =

a. 4,325,270 ☐ 4,246,370

b. 3,507,469 ☐ 3,206,986

c. 5,651,845 ☐ 5,461,835

d. 2,453,578 ☐ 2,453,587

e. 9,678,450 ☐ 9,768,675

7. Count in hundred-thousands and list the numbers. The first one is done for you.
   a. From 124,000 to 524,000

      124,000, 224,000, 324,000, 424,000, 524,000
   b. From 230,000 to 930,000
   c. From 376,000 to 776,000
8. Count in millions and list the numbers.
   a. From 1,250,000 to 6,250,000
   b. From 4,600,000 to 9,600,000
9. Order these numbers

   423,635 947,534 3,604,376 837,209 5,628,370

   480,982 408,893 469,743 6,086,304 873,276

1.1.4 Even and Odd Whole Numbers

Remember that you have learnt about Even and Odd whole numbers in your previous mathematics lessons. **Even numbers** end in 0,2,4,6 and 8, and **odd numbers** end in 1,3,5,7 and 9.

Activity 1.4

1. List even numbers between 1,253,401 and 1,253,411
2. List odd numbers between 2,430,678 and 2,430,688
3. Complete
   a. 3,570,602 3,570,604 3,570,606 ________ ________ ________ ________ ________ 3,570,618
   b. 6,620,403 6,620,405 6,620,407 ________ ________ ________ ________ ________ 6,620,419

4. Find the number.
   a. I am an even number. I come between 2,438,670 and 2,438,674. What number am I?
   b. I am an odd number. I come between 3,156,257 and 3,156,261. What number am I?
5. Use seven small pieces of card (or paper) numbered 1 to 7

Figure 1.7

   a. Write 4 even numbers with three digits.
   b. Write 4 odd numbers with four digits.

Here you will learn about properties of even and odd numbers in more detail.

Study the following examples:

Example 6

**a) Both 23 and 45 are odd numbers. But their sum:
23 + 45 = 68 is an even number.**

**b) Both 32 and 54 are even numbers. Also their sum:
32 + 54 = 86 is an even number.**

**c) Consider the sum of an even number and an odd number:
32 + 45 = 77 is an odd number.**

Group work 1.3

What can you conclude about

Even + Even?
Odd + Odd?
Even + Odd?

Exercise 1.D

Determine whether each of the following statements is true or false.

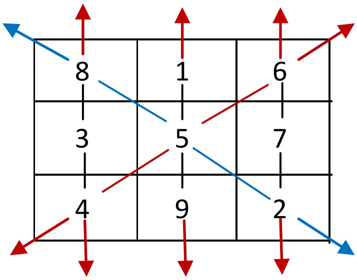
1. The sum of two even numbers is even.
2. The sum of two odd numbers is odd.
3. The sum of an odd number and an even number is an even number.
4. An even number is divisible by 2.
5. If a number ends in 7, then it is odd.
6. The sum of three odd numbers is odd.
7. The sum of any five whole numbers is odd.
8. The sum of any four consecutive whole numbers is even.

1.2 Operation on Whole Numbers

1.2.1 Addition and Subtraction of Whole numbers

Activity 1.5

In a magic square, the sums of the numbers in every row, column, and diagonal are equal.



8	1	6
3	5	7
4	9	2

a	71	b
99	c	155
113	183	85

Figure 1.8

What are the missing numbers?

You know that addition, subtraction, multiplication and division are four fundamental operations of mathematics. Here, you will learn about the properties of these operations on whole numbers.

Example 7

a. What is the missing number in

4 + ☐ = 7 and ☐ + 4 = 7?

Did you answer "3"? That's good. What do you notice then?

4 + 3 = 3 + 4 = 7

Do you remember the commutative property of addition?

a + b = b + a for two whole numbers a and b.

b. Consider the sum (3 + 4) + 5 and 3 + (4 + 5). Is the sum equal? Do you remember the associative property of addition?

(a + b) +c = a +(b + c) for three whole numbers a, b, and c

c. What is the missing number in both

☐ + 4 = 4 and 4 + ☐ = 4?

Do you remember the property of 0 on whole numbers?

x + 0 = 0 + x = x for a whole number x.

Do the above three properties hold true for subtraction? Why?

When we add numbers we need to keep the digits in the correct columns, and take care with grouping and regrouping. Here are some examples. Study the examples carefully.

Example 8

$$\begin{array}{r} 6\ 2\ 4\ 2\ 3\ 6 \\ +\ \underline{1\ 3\ 3\ 4\ 9\ 5} \\ 7\ 5\ 7\ 7\ 3\ 1 \end{array}$$

Step 1. 6 and 5 ones are 11 ones. 11 ones make 1 Ten and 1 one. Write 1 carry ten.

Step 2. 1,3 and 9 Tens are 13 Tens. 13 Tens make 1 hundred and 3 Tens. Write 3 carry 1 Hundred.

Step 3. 1,2 and 4 Hundreds are 7 Hundreds. Write 7.

Step 4. 4 and 3 Thousands are 7 Thousands. Write 7.

Step 5. 2 and 3 Ten Thousands make 5 Ten Thousands. Write 5.

Step 6. 6 and 1 Hundred Thousands make 7 Hundred Thousands. Write 7.

Example 9

$$\begin{array}{r} 3\ 3\ 4\ 2\ 9\ 7 \\ +\ \underline{4\ 9\ 5\ 9\ 6\ 8} \\ 8\ 3\ 0\ 2\ 6\ 5 \end{array}$$

Step 1. 7 and 8 ones are 15 ones. Write 5. Carry 1 Ten.

Step 2. 1, 9 and 6 Tens are 16 Tens. Write 6. Carry 1 Hundred.

Step 3. 1,2 and 9 Hundreds are 12 Hundreds. Write 2, carry 1 Thousand.

Step 4. 1,4 and 5 Thousands are 10 Thousands write 0. Carry 1 Ten Thousands.

Step 5. 1,3 and 9 Ten Thousands make 13 Ten Thousands. Write 3. Carry 1 Hundred Thousands.

Step 6. 1,3 and 4 Hundred Thousands make 8 Hundred Thousands. Write 8.

Group work 1.4

Add

a. $$\begin{array}{r} 8\ 2\ 4\ 6\ 0\ 8 \\ +\ \underline{3\ 4\ 7\ 7\ 6\ 5} \\ \underline{} \end{array}$$

b. $$\begin{array}{r} 9\ 3\ 3\ 4\ 8\ 7 \\ +\ \underline{6\ 7\ 8\ 3\ 2\ 5} \\ \underline{} \end{array}$$

Note that Subtraction is the reverse process of addition.

$$\begin{array}{r} 83\ 02\ 65 \\ -\ \underline{33\ 42\ 97} \\ 49\ 59\ 68 \end{array} \quad \text{and} \quad \begin{array}{r} 83\ 02\ 65 \\ -\ \underline{49\ 59\ 68} \\ 33\ 42\ 97 \end{array}$$

Example 10

$$\begin{array}{r} 56542 \\ -\ \underline{36886} \\ 19656 \end{array}$$

Step 1. 2 ones, take away 6, I can't. Take 1 Tens leaving 3. Change it to 10 ones, 12 ones, take away 6 is 6. Write 6.

Step 2. 3 Tens, take away 8, I can't. Take 1 hundred leaving 4. Change it to 10 Tens. 13 Tens, take away 8 is 5. Write 5.

Step 3. 4 Hundreds, take away 8, I can't. Take 1 Thousands leaving 5. Change it to 10 Hundreds. 14 Hundreds, take away 8 is 6. Write 6.

Step 4. 5 Thousands, take away 6, I can't, take 1 Ten Thousands leaving 4. Change it to 10 Thousands. 15 Thousands, take away 6 is 9, write 9.

Step 5. 4 Ten Thousands, take away 3, is 1, write 1.

Note that

$$\begin{array}{r} 56542 \\ -\underline{19656} \\ \mathbf{36886} \end{array} \quad \text{and} \quad \begin{array}{r} 36886 \\ +\underline{19656} \\ \mathbf{56542} \end{array}$$

Activity 1.6

1. Write the missing numbers.

   a. 8 + ☐ = 8

   b. ☐ + 9 = 9

   c. 10 - ☐ = 10

   d. (6 + 7) + ☐ = 6 + (7 + 8)

   e. (300 + 500) + 600 = 300 + (500 + ☐)

   f. 2,456 + 3,580 =3,580 + ☐

2. Complete. The first one is done for you.

a.
$$\begin{array}{r} 3\ 8 \\ \underline{+\,5\ 4} \\ \boxed{9}\boxed{2} \end{array} \qquad \begin{array}{r} \boxed{9}\boxed{2} \\ \underline{-\,3\ 8} \\ \boxed{5}\boxed{4} \end{array} \qquad \begin{array}{r} \boxed{9}\boxed{2} \\ \underline{-\,5\ 4} \\ 3\ 8 \end{array}$$

b.
$$\begin{array}{r} 4\ 3\ 2 \\ \underline{+\,2\ 6\ 9} \\ \boxed{}\boxed{}\boxed{} \end{array} \qquad \begin{array}{r} \boxed{}\boxed{}\boxed{} \\ \underline{-\,2\ 6\ 9} \\ \boxed{}\boxed{}\boxed{} \end{array} \qquad \begin{array}{r} \boxed{}\boxed{}\boxed{} \\ \underline{-\,4\ 3\ 2} \\ \boxed{}\boxed{}\boxed{} \end{array}$$

c.
$$\begin{array}{r} 5\ 4\ 4\ 5 \\ \underline{+\,2\ 3\ 8\ 7} \\ \boxed{}\boxed{}\boxed{}\boxed{} \end{array} \qquad \begin{array}{r} \boxed{}\boxed{}\boxed{}\boxed{} \\ \underline{-\,5\ 4\ 4\ 5} \\ \boxed{}\boxed{}\boxed{}\boxed{} \end{array} \qquad \begin{array}{r} \boxed{}\boxed{}\boxed{}\boxed{} \\ \underline{-\,2\ 3\ 8\ 7} \\ \boxed{}\boxed{}\boxed{}\boxed{} \end{array}$$

3. If we add any two whole numbers a and b, is it true that the new number is also a whole number?

4. Write > , < or = in the box in order to compare.

a. 232,567 + 687,758 ☐ 354,743 + 467,869

b. 358,676 + 576,589 ☐ 2,121,342 + 3,436,536

c. 6,234,238 – 4,867,786 ☐ 7,158,349 - 3,283,898

Let us deal with solving word problems related to real life.

Example 11

A book has 1549 letters on the first page, 1672 on the second page and 1847 on the third page. What is the total number of letters on the first three pages?

Solution:

$$\begin{array}{r} 1549 \\ +\ 1672 \\ \underline{1847} \\ 5068 \end{array}$$

There are 5,068 letters on the first three page

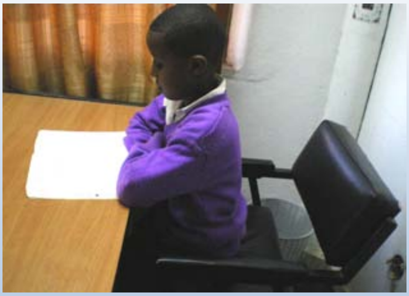


Figure 1.9

Example 12

A woman has Birr twenty three thousand, eight hundred forty but had to pay Birr two thousand five hundred seventy five for some clothes. How much did she have left?

Solution:

$$\begin{array}{r} 2\,3\,8\,4\,0 \\ -\ \underline{2\,5\,7\,5} \\ 2\,1\,2\,6\,5 \end{array}$$

She was left with Birr 21,265.

Exercise 1. E

1. Add or subtract.

a) $\begin{array}{r} 43257 \\ \underline{+\ 15894} \\ \underline{\qquad\quad} \end{array}$

b) $\begin{array}{r} 56674 \\ \underline{+\ 48486} \\ \underline{\qquad\quad} \end{array}$

c) $\begin{array}{r} 727585 \\ \underline{+\ 575869} \\ \underline{\qquad\quad} \end{array}$

d) $\begin{array}{r} 94328 \\ -\ 56779 \\ \hline \\ \hline \end{array}$

e) $\begin{array}{r} 79024 \\ -\ 68968 \\ \hline \\ \hline \end{array}$

f) $\begin{array}{r} 810731 \\ -\ 799843 \\ \hline \\ \hline \end{array}$

2. A ship carries 8,754 bags of cocoa and 1,296 bags of coffee. How many bags are there altogether?
3. A large farm had seven thousand seven hundred cattle. They bought one thousand, five hundred seven more cattle. How many cattle did the farm have altogether?

Figure 1.10

4. The number of people in three towns are 12,542, 11,460 and 13,627. What is the total population of all the three towns?
5. In a factory where eight thousand, four hundred thirty two people worked, four thousand, nine hundred seventy one were men. How many women worked at the factory?
6. The male population of Ethiopia in the year 2007 (according to CSA) was 37,296,657 and the female population was 36,621,848.
   a. Which was the larger population- male or female?
   b. What was the total population of Ethiopia?
   c. Find the difference between female and male populations.

Figure 1.11

7. A man had Birr 1,052,747 in his bank account. If he withdrew Birr 905,002 and Birr 87,445 in two consecutive months, then how much money was left in his account?
8. In one year, 33,000,000 boxes of lemons and limes were produced. 1,200,900 boxes were limes. How many boxes of lemons were there?

1.2.2 Multiplication of Whole Numbers

Activity 1.7

One packet contains 6 pencils. How many pencils are in 3 packets?

What is 3 × 6?

Is 6 × 3 the same as 3 × 6?

3 + 3 + 3 + 3 + 3 + 3 =_____

and 6 + 6 + 6 = _______

What do you conclude?

Figure 1.12

Remember that multiplication is a repeated addition. You have learnt how to multiply two natural numbers. In this section you will study some properties of multiplication on whole numbers in more detail.

Do you remember?

1. Multiplication of numbers is **commutative**. That is, if a and b are whole numbers, then a × b = b × a.

   Does the **associative property** apply to multiplication?

   Multiply 2 × 3 × 5.

   $2 \times 3 \times 5 = (2 \times 3) \times 5$ or $2 \times 3 \times 5 = 2 \times (3 \times 5)$

   $= 6 \times 5$ $= 2 \times 15$

   $= 30$ $= 30$

2. The **associative property** also applies to multiplication. That is, if a, b and c are three whole numbers, then $(a \times b) \times c = a \times (b \times c)$.
3. Look at this multiplication.

   $25 \times (10 + 2) = 25 \times 12 = 300$

Is it true that 25 × (10 + 2) = (25 × 10) + (25 ×2)?

(25 × 10) + (25 × 2) = 250 + 50= 300

Cheek with 45 × 8

Is it true that 45 × 8 = (40 + 5) × 8 ?

$$\begin{aligned}(40 + 5) \times 8 &= (40 \times 8) + (5 \times 8)\\ &= 320 + 40\\ &= 360\end{aligned}$$

This is called the **distributive property of multiplication over** addition. That is, if a, b and c are three whole numbers, then

$$a \times (b + c) = (a \times b) + (a \times c)$$

4. You have seen that 2 × 1 = 1 × 2 and also 2 × 1 = 2 and 1 × 2 = 2. Observe that any whole number multiplied by 1 stays the same. That is, if a is a whole number, then a × 1 = 1 × a = a
5. **Multiplication property of 0** is given below:

   4 × 0 = 0 × 4 and also 4 × 0 = 0 and 0 × 4 = 0.

   Here we understand that any number multiplied by zero equals zero. That is, if a is a whole number, then a × 0 = 0 × a = 0

Group work 1.5

Tigist's heart rate is 78 beats per minute. Almaz's heart rate is 80 beats per minute. How many times do their heart beat altogether in 3 minutes?

The following example discusses the use of the distributive property. Study the example carefully

Example 13

$$\begin{aligned}3{,}457 \times 28 &= 3{,}457 \times (20 + 8)\\ &= (3{,}457 \times 20) + (3{,}457 \times 8)\\ &= (3{,}457 \times 2 \times 10) + (27{,}656) && \text{(Why?)}\\ &= (6{,}914 \times 10) + 27{,}656 && \text{(Why?)}\\ &= 69{,}140 + 27{,}656\\ &= 96{,}796\end{aligned}$$

Activity 1.8

Complete

a. 4,326 × 15 = 4,326 × (10 + 5)
= (4,326 × 10) + (4,326 × □)
= (□) + (□)
= □

b. 3,674 × 28 = 3,674 × (□ + □)
= (3,674 × □ × 10) + (3674 × □)
= (□ × 10) + □)
= (□ + □)
= □

c. 4,318 × 34 = 4,318 × (□ + □) = (4,318 × □ × 10) +
(4,318 × □)
= (□ × 10) + □ = □ + □ = □

d. 7,508 × 63 = 7,508 × (□ + □)
= (7,508 × □ × 10) + (7,508 × □)
= (□ × 10) + □
= □ + □
= □

Study the steps when multiplying two whole numbers in the following example.

Example 14

Multiply

a.
$$\begin{array}{r} 287 \\ \underline{\times\quad 3} \\ 861 \end{array}$$

Step 1. 3 × 7 ones = 21. Write 1 and carry 2 Tens.

Step 2. 3 × 8 Tens = 24.
24 + 2 = 26. Write 6 and carry 2 hundreds.

Step 3. 3 × 2 Hundreds = 6
6 + 2 = 8

b.
$$\begin{array}{r} 457 \\ \underline{\times 28} \\ 3656 \\ \underline{9140} \\ 12796 \end{array}$$

Step 1. 8 × 7 = 56, write 6, carry 5 Tens.

Step 2. 8 × 5 = 40, 40 + 5 = 45

Step 3. 8 × 4 = 32 hundreds

Multiply by 10: write 0. Then multiply by 2.

Step 4. 2 × 7 = 14 Tens, write 4, carry 1 Hundred.

Step 5. 2 × 5 = 10 Hundreds. 10 + 1 = 11, write 1, carry 1 thousand.

Step 6. 2 × 4 = 8 Thousands. 8 + 1 = 9. write 9
3656 + 9,140 = 12,796

Example 15

A store rents space in a building at a cost of Birr 20 per square meter. If the store is 700 square meter, how much is the rent?

Solution

20 × 700
= 14,000
Therefore, the rent is Birr 14,000

Note that an **estimate** can indicate the size of a product. The following example discusses about working with approximate values for determining rough estimation when multiplying large numbers. Study the example carefully.

Example 16

6127 × 294 ≈ 6000 × 300 = 1,800,000 (rounding 6127 to thousands and rounding 294 to hundreds respectively)

6127 × 294 = 1,801,338 ≈ 1,800,000

Exercise 1.F

1. Multiply

   a. $\begin{array}{r} 14 \\ \times\ 2 \\ \hline \end{array}$
   d. $\begin{array}{r} 168 \\ \times\ 5 \\ \hline \end{array}$
   g. $\begin{array}{r} 804 \\ \times\ 93 \\ \hline \end{array}$

   b. $\begin{array}{r} 23 \\ \times\ 3 \\ \hline \end{array}$
   e. $\begin{array}{r} 63 \\ \times\ 14 \\ \hline \end{array}$
   h. 204 × 32 = ☐
   i. 743 ×25 = ☐

   c. $\begin{array}{r} 36 \\ \times\ 7 \\ \hline \end{array}$
   f. $\begin{array}{r} 571 \\ \times\ 28 \\ \hline \end{array}$
   j. 250 × 12 × 6 = ☐
   k. 304 × 31 × 8 = ☐

2. Estimate the product
   a. 2112 × 198
   b. 3104 × 395

3. Fatuma bought 3 baskets of Mangoes. There were 25 Mangoes in each basket. How many Mangoes did she have altogether?

Figure 1.13

4. A school-week has 5 days. How many school-days are there in 42 weeks?
5. A Scientific dictionary has 1,236 pages. How many pages would 24 such dictionaries have?
6. Each day a man sells 3,762 copies of a news paper. How many copies can the man sell in two months?
7. A factory produced 483 motor bikes in a year. If the profit on one bike is Birr 5,830, how much profit did the factory make during the year?

1.2.3 Division of Whole Numbers

Look at the following divisions

(i) $6 \div 2 = 3$, remainder = 0

(ii) $8 \div 3 = 2$, remainder = 2

(iii) $9 \div 3 = 3$, remainder = 0

(iv) $6 \div 8 = ?$ here quotient is not a whole number.

In case of (i) and (iii), you can see that remainder is 0, i.e., one whole number completely divides another whole number and the result is a whole number.

In case of (ii) and (iv) when one whole number divides another whole number, the result is not a whole number.

Activity 1.9

Determine the quotient and remainder

a. $12 \div 3$

b. $13 \div 4$

c. $18 \div 2$

d. $16 \div 5$

Study the examples given below on division carefully.

Example 17

A box contains 56 shirt buttons. If a shirt needs 7 buttons, how many shirts can be made up from the box?

Solution: 56 ÷ 7 = 8 because 7 × 8 = 56

8 shirts can be made up from the box.

Example 18

a. 24 ÷ 8 = 3 because 8 × 3 = 24
b. 60 ÷ 5 = 12 because 5 × 12 = 60
c. 6000 ÷ 3 = 2000 because 3 × 2000 = 6000

you may also use the long division to divide numbers.

Example 19

$$\begin{array}{r|l} & 1\,3\,2 \\ \hline 7 & 9\,2\,4 \\ & \underline{-7} \\ & 2\,2 \\ & \underline{2\,1} \\ & 1\,4 \\ & \underline{1\,4} \\ & 0 \end{array}$$

Reminder

9 hundreds ÷ 7=1 hundred, write 1 above the 9 in the hundreds column.

7 × 1 hundred= 7 hundreds, write 7 under the 9.9-7=2. Bring down the 2 Tens.

22 tens ÷ 7=3 tens. write 3 above the 2 in the tens column,

7 × 3 Tens = 21, write 21 under the 22. 22 – 21 = 1. Bring down the 4 units. 14 ÷ 7 = 2 units write 2 above the 4 in the units column 7 × 2 units = 14 units, write 14 under the 14. 14-14 = 0.

The answer or quotient is 132.

Do you remember?

- In any division

Dividend = (quotient) (divisor) + remainder

- 0 ÷ a = 0 if a is a non-zero whole number.
- a ÷ 1 = a for any whole number a.
- Division is not commutative as well as associative (Why?)

Example 20

6 6
15)1000
9 0
1 0 0
9 0
1 0

Quotient = 66
Remainder = 10
Check that 1000
= 66 × 15 + 10

1 thousand ÷ 15? I can't.
10 hundreds ÷ 15? I can't.
100 tens ÷ 15 is 6 tens.
Write 6 in the quotient's Tens column. 15 × 6 = 90.
Write 90 under the 100.
100 - 90 = 10. Bring the 0 units down, 100 units ÷ 15 is 6.
Write 6 in the quotient's units column. 15 × 6 = 90. Write 90 under the 100.
100 - 90 = 10
The quotient is 66. The remainder is 10

Example 21

Divide 1,801,340 by 294

Check that

1,801,340 = 6127 × 294 + 2

6127 ← quotient
294)1,801, 340
1764
373
294
794
588
2060
2058
2 ← remainder

Exercise 1.G

1. Divide and check by multiplying. Write the quotient and remainder in each case.

a. 197 ÷ 6
b. 216 ÷ 5
c. 639 ÷ 7
d. 876 ÷ 9
e. 908 ÷ 15
f. 800 ÷ 27
g. 43,567 ÷ 372
h. 67,890 ÷ 124
i. 278,056 ÷ 6072

2. Complete

a ÷ b = 3	a	18	27	36		102		9000
	b	6			20		100	

3. How many weeks are there in 5887 days?
4. How many days are there in 360 hours?
5. There are 2400 eggs that are to be shared equally in to 96 groups. How many eggs must each group get?

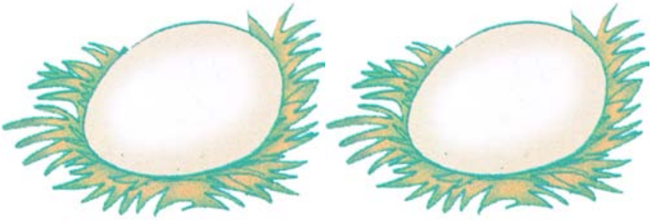


Figure 1.14

6. Three hundred eighty children share 8120 oranges. How many oranges will each child get? How many oranges are left over?
7. Find the number which when divided by 36 gives 352 as the quotient and 27 as the remainder.

1.2.4 Problems Containing Several Operations

A numerical expression is made up of numbers and operations. When simplifying a numerical expression, rules must be followed so that everyone gets the same answer.

Activity 1.10

Use two or more operations with these digits to make totals.

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$ $\boxed{7}$ $\boxed{8}$ $\boxed{9}$

a. Total = 15

$15 = (2 + 3) \times 3$ using + and ×

$15 = 3 \times (7 - 4) + 6$ using +, - and ×

$15 = (6 \div 3) \times (5 + 3) - 1$ using +, - , × and ÷

$15 = (8 \div 4) \times (6 + 2) - 1$ using +, - , × and ÷

b. Use two or more operations and the numbers from 1 to 9 to make total = 25

Definition 1.1: A numerical expression is made up of numbers and operations. When simplifying a numerical expression, rules must be followed so that everyone gets the same answer.

From your previous mathematics lessons, observe that we use the four operations (+, -, ×, ÷) in this way:

Solve what is in the bracket first, followed by 'of', then division, multiplication, addition and subtraction (BODMAS).

Group work 1.6

A student simplified $8\times(9 + 13)$ as follows:

$$8\times(9 + 13) = 8 \times 9 + 13$$
$$= 72 + 13$$
$$= 85$$

What is the student's error?

Example 22

Evaluate

a) $\dfrac{9+1\times 6}{(1+4)\times 3}+5$

b) $\dfrac{43\times 2\times 3-33}{25\times 3}$

c) $\dfrac{(6+100)-25}{3\times 3}$

d) $2\times 9\div 3-1$

Solution:

a) $\dfrac{9+1\times 6}{(1+4)\times 3}+5=\dfrac{9+6}{5\times 3}+5$

$=\dfrac{15}{15}+5=1+5=6$

b) $\dfrac{43\times 2\times 3-33}{25\times 3}=\dfrac{258-33}{75}$

$=\dfrac{225}{75}=3$

c) $\dfrac{(6+100)-25}{3\times 3}=\dfrac{106-25}{9}$

$=\dfrac{81}{9}=9$

d) $2\times 9\div 3-1=18\div 3-1$

$=6-1=5$

Exercise 1. H

1. Identify whether each of the following statements is true or false.

   a. $4\times(20-10)=(7\times 5)+5$

   b. $(27\div 9)+9>27\div(3+6)$

   c. $(20\div 2)\div 2<20-(18\div 3)$

   d. $(3\times 4)+(3\times 5)<(4\times 3)+(5\times 3)$

   e. $(32\div 4)+(36\div 4)=(4\times 2)+(3\times 3)$

   f. $(4\times 7)-(20-10)>(7\times 4)-(10-5)$

   g. $(25\div 5)+10>(30\div 6)+15$

   h. $(36\div 2)-15=(30\div 2)-12$

   i. $(5\times 4)+(6\times 4)<(6\times 3)+(7\times 3)$

   j. $(18\div 3)\times(30\div 5)=(6\times 6)\div(3\times 2)$

Figure 1.15

2. Calculate the value of the following. The first one is done for you
   a. $324 + (512 - 473) \div 3$
      $= 324 + 39 \div 3$ because $512 - 473 = 39$
      $= 324 + 13$ $39 \div 3 = 13$
      $= 337$
   b. $285 + (483 - 387) \div 4$
   c. $(5000 - 800) \div 70 + 23$
   d. $16 \times (24 \div 4) + 10$
   e. $(5 \times 4 + 4) \div (4 \times 4 - 8)$
   f. $(15 \times 2) \div (14 + 1)$
   g. $100 - (12 \div 4 + 2)$

1.2.5 Multiples and Divisors of Whole Numbers

Activity 1.11

1. Write each number as a product of two whole numbers in as many ways as possible
   (a) 6 (b) 16 (c) 17 (d) 36
2. Amare bikes every third day and walks every other day. On Meskerem 5, Amare biked and walked. When will he do both again?

The **divisors (factors)** of a number are all those numbers which will divide into that number with no remainder.

Example 23

a. 24 can be divided by 24,12,8,6,4,3,2,1. So the divisors (factors) of 24 are 24,12,8,6,4,3,2 and 1.
b. 1,2,3,5,6,10,15 and 30 are divisors of 30.

The multiples of a number are found by multiplying the number by 0,1,2,3,4,---

Example 24

Some of the multiples of 6 are:

$6 \times 0 = 0$, $6 \times 1 = 6$, $6 \times 2 = 12$, $6 \times 3 = 18$ $6 \times 4 = 24$

0,6,12,18 and 24 are multiples of 6.

What are some other multiples of 6?

Exercise 1.I

1. List all multiples of 5 less than 62.
2. List all multiples of 7 between 20 and 60.
3. What are multiples of 8?
4. Write down divisors of 32?
5. Write down common divisors of 18 and 32.

1.2.6 Power of Whole Numbers

Activity 1.12

Write as in the first example shown below

a) $2\times2\times2=2^3$

b) $3\times3\times3\times3=$

c) $4\times4\times4\times4\times4=$

When we write, $2\times2\times2\times2\times2$ as 2^5, read as **two raised to power five** or simply **two raised to five**. We know $2^5 = 32$ because $2\times2\times2\times2\times2 = 32$. Here 2 is called the **base** and 5 is called the **exponent.**

Example 25

a. $3^4 = 3\times3\times3\times3 = 81$

b. $5^3 = 5\times5\times5 = 125$

5^3 may be read as five cubed

c. $4^2 = 4 \times 4 = 16$

4^2 may be read as four squared

$$\underset{\text{base}}{4}^{\overset{\text{Exponent}}{2}}$$

Study the following example:

Example 26

a. **We may write $2^3 \times 2^4$ as 2^{3+4} or 2^7 because $2^3 \times 2^4$**

$$= 2\times2\times2\times2\times2\times2\times2 = 2^7$$

b. $\frac{3^6}{3^4}$ **may be written as 3^{6-4} or 3^2 because**

$$\frac{3^6}{3^4} = \frac{3\times3\times3\times3\times3\times3}{3\times3\times3\times3}$$

$$= \frac{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}} \times 3 \times 3 = 3^2 = 9$$

Group work 1.7

Which numerical expression simplifies to 77?

(a) $3^2 \times (4 + 5)$

(b) $7 + 4^3 + 10$

(c) $3 \times 5^2 + 2$

(d) $10^2 - 4 \times 5 + 1$

Note: When you evaluate a numerical expression, which involves power of whole numbers, you need to follow the following rules:

Order of operations:

1. Do all operations within grouping symbols first; start with the inner most grouping symbols.
2. Do all powers before other operations.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Example 27

Perform the indicated operations $(14 + 16) \div 5 \times 4 + (4^3 - 4)$

Solution:

$(14 + 16) \div 5 \times 4 + (4^3 - 4)$ here $14 + 16 = 30$, $64-4=60$ and $30\div5=6$

$= 30 \div 5 \times 4 + (64 - 4)$

$= 6 \times 4 + 60$

$= 24 + 60 = 84$

Exercise 1.J

1. Write the following numbers in power form. The first one is done for you.

a. $64 = 2\times2\times2\times2\times2\times2 = 2^6$

b. 27 = ________________ = ____

c. 32 = ________________ = ____

d. 125 = _______________ = ____

e. 1000 = ______________ = ____

2. Find the value of x, if
   a. $x^3 = 8$ (Example (a) if $x^3 = 8$, then $x^3 = 8 = 2\times2\times2 = 2^3$.
      Therefore $x = 2$).
   b. $x^3 = 27$
   c. $x^3 = 125$
   d. $x^3 = 1000$
3. Complete the table

Number	**8**	**9**	**16**	**25**	**32**	**64**	**81**
a^n	2^3						
Exponent	3		4	2		3	
Base	2	3			2		3

4. Compare using >, < or =

a. 2^3 ——— 3^2

c. 2^5 ——— 5^2

b. 4^3 ——— 3^4

d. 2^{10} ——— 10^2

5. Complete

Number	**Product of Sevens**	**Number of Sevens**	**Number using exponents**
7	7	1	7
49	7×7		
	7×7×7		
2,401			
			7^5
	7×7×7×7×7×7		
		7	
5,764,801			

6. Evaluate

a) $\frac{36}{3^2 - 3}$

b) $(5^2 + 3) \div 7$

c) $(20 + 30) \div 5 \times 2 + (2^4 - 1)$

UNIT SUMMARY

Important facts you should know:

- One **Million** (1,000,000) is a seven digit number.
- Any whole number n different from zero has a **predecessor** "n – 1" and a **successor** "n + 1".
- **Place value chart**

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1,000000	100,000	10,000	1,000	100	10	1

$8,574,629 = (8 \times 1,000,000) + (5 \times 100,000) + (7 \times 10,000) + (4 \times 1,000) + (6 \times 100) + (2 \times 10) + 9$

- **Even numbers** end in 0,2,4,6 and 8 and **odd numbers** end in 1,3,5,7 and 9.
- $x + 0 = 0 + x = x$ for a whole number x.
- $a \times b = b \times a$ for whole numbers a and b.
- $(a \times b) \times c = a \times (b \times c)$ for whole numbers a, b and c.
- In any division Dividend = (quotient) (divisor) + remainder.
- We use the four operations (+, -, ×, ÷) in this way: **BODMAS**.
- The **divisors (factors)** of a number are all those numbers which will divide into the number with no remainder.
- When we write, $2 \times 2 \times 2 \times 2 \times 2$ as 2^5 raised to five), $2^5 = 32$. (2 is called the base and 5 is called the exponent).
- We may write $2^4 \times 2^5$ as 2^{4+5} or 2^9.

REVIEW EXERCISE

1. Write these numbers in words.
   a. 4,350,672 ______________________________
   b. 7,582,091 ______________________________
   c. 10,093,385 ______________________________
   d. 16,724,109 ______________________________
   e. 20,000,000 ______________________________
   f. 83,000,400 ______________________________
2. Write these numbers in figures.
   a. Seven million, ten thousand eighty six.
   b. Twelve million, seven hundred thousand, one hundred three.
   c. Fourteen million, sixteen.
   d. Thirty seven million, six hundred twenty five thousand, forty nine.
3. a. What is the predecessor of 5,907,183?
   b. What is the successor of 7,068,439?
   c. What is the predecessor of 8,907,056?
   d. What is the successor of 12,000,400?
4. Compare the numbers using >, < or =
   a. 3, 5 8 6, 2 7 5 ☐ 3, 6 5 8, 7 5 2
   b. 10, 7 0 6, 0 0 9 ☐ 10, 0 9 9 , 9 9 1
   c. 13, 2 1 8, 7 8 0 ☐ 13, 9 0 0, 0 0 0
   d. 21, 0 0 7, 7 0 0 ☐ 21, 0 0 8, 0 0 0
   e. 38, 7 0 4, 1 0 0 ☐ 38, 4 0 7,1 0 0
5. Write the place value of 8 in the number 13,826,004.
6. Write the following numbers in expanded form.
   a. 21,706,489
   b. 34,069,705
   c. 91,360,072

7. Write the whole number which is represented by the following expanded form.
   a. (4 × 1,000,000) + (7 × 10,000) + (5 × 1,000) + (9 × 10) + 1
   b. (7 × 1,000,000) + (9 × 100,000) + (6 × 100) + (8 × 10)
   c. (9 × 1,000,000) + (8 × 100) + (6 × 10) + 3
   d. (6 × 10,000,000) + (7 × 1,000,000) + (7 × 1000) + 9

8. Count in millions and list the numbers.
   a. From 1,300,200 to 8,300,200
   b. From 13,407,500 to 20,407,500
   c. From 30,566,409 to 39,566,409

9. a. List even numbers between 30,708,969 and 30,708,983.
   b. List odd numbers between 42,561,842 and 42,561,852.

10. a. What is the sum of three even numbers? (Even, Odd)
    b. What is the sum of four odd numbers? (Even, Odd)
    c. What is the sum of five odd numbers? (Even, Odd)

11. Add

a. 8,346,271
+ 4,077,956

b. 13,097,805
+ 7,903,769

c) 24,681,967
+18,098,123

12. Subtract

a. 18,076,045
- 6,953,852

b. 21,606,909
- 8,079,098

c) 32,168,432
- 9,969,909

13. Multiply

a. 3468
× 94

b. 7086
× 29

c) 9431
× 573

14. Divide

a. $576,262 \div 73$ c. $3,008,916 \div 6042$

b. $3,945,305 \div 845$ d. $6,352,731 \div 927$

15. Perform the indicated operations

a. $4257 + (6028 - 5993) \div 5$

b. $250 \times (300 \div 6) + 150$

c. $(420 \times 6 + 4) \div (16 \times 2 - 28)$

d. $4^3 - 2 \times 5 + (8 \div 2)$

e. $[(4 + 12 \div 4) -2]^3$

16. Write in power form.

a. 243 c. 2401

b. 128 d. 625

17. Zeberga bought two tickets for the instant lottery and still had Birr 85,234 in the bank. He dreamt that he had a winning ticket worth Birr 750,000 and another worth Birr 480,000. How much money would Zeberga have if his dream was **true?**

18. Ato Wondimu was the head master of a primary school in Holeta. He had Birr 854,550 in school fund. He paid his teachers' salaries and then had a total of Birr 45,680 left. How much did he have to pay the teachers?

19. In a school hall there are 1432 benches. Each bench can hold 16 children. How many children can sit on the benches in the hall?

20. Ato Dinkessa and Woizero Fatuma run a library. They have 32,448 books altogether. They ask a class of 52 students to carry the books to a new room. If each students carries the same number of books, how many will each of them carry?

UNIT 2

WORKING WITH VARIABLES

Unit Outcomes: After completing this unit, you should be able to:

- realize the use of variables in Mathematics.
- understand Mathematical terms, expressions and simplification of expressions.
- identify equations and inequalities and determine their value by substitution.

Introduction

In earlier grades, you have been dealing with the numbers 0,1,2,3,4, etc. You have used the four fundamental operations to do mathematical calculations. In the present unit, you will be introduced to algebraic terms, values of terms, and values of simple algebraic expressions. You shall also learn about equations and inequalities solved by substitution.

2.1 Algebraic Terms and Expressions

This section introduces some basic concepts and expressions used in algebra. Solving real-world problems is an important part of algebra, so you will be introduced with algebraic terms and mathematical expressions that often arise in applications.

2.1.1 Algebraic Terms and Values of terms

Do you recall, in arithmetic, that you have been dealing with the numbers 0, 1, 2, 3, 4, 25, 36, 100 etc? You have used the four fundamental operations (+, -, ×, ÷) to do mathematical calculations.

Activity 2.1

Tell the operation in the expression

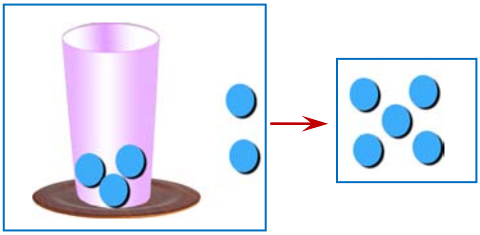
a) $15 - y$ b) $4x$ c) $z \div 6$ d) $x + y$ e) x^2

Probably the greatest difference between arithmetic and algebra is the use of **variabels** in algebra. When a letter represents a number, that letter is a **variable**. Study the following explanation:

The phrase **the sum of two and some number** is an algebraic expression. This phrase contains a **constant** that you know, 2, and an unknown value "some number".

- You can use counters to represent 2 and a cup to represent the unknown value.

- Any number of counters may be in the cup. Suppose you put 3 counters in the cup. Instead of an unknown value, you know the cup has a value of 3. When you empty the cup and count all the counters, the expression has a value of 5.



- Consider the phrase **three times some number**. Since you don't know the value of the number, let a cup represent this value. Since it is three times some number, you will need to use three cups. The same number of counters should be in each cup.

Figure 2.1

Activity 2.2

Work in group

Model each phrase with cups and counters. Then put five counters in each cup. How many counters are there in all? Record your answer by drawing pictures of your models.

1. the sum of 7 and a number
2. twice a number
3. 5 more than a number
4. six times a number

- Can you write a sentence to describe what the cup represents? Write a sentence that explains why x+4 is called an algebraic expression.

Study the following example.

Example 1

Yeshi charges Birr 4 for selling a bottle of soft drink. If she sells one bottle of soft drink, she makes 1×4 or Birr 4. If she sells two bottles of soft drink, she makes 2×4 or Birr 8. The amount she makes increases with the number of bottles of soft drink she sells.

You can make a table to show the pattern between the number of bottles of soft drink sold and the amount earned.

Number of bottles	Amount Earned
0	Birr 4×0 = Birr 0
1	Birr 4×1 = Birr 4
2	Birr 4 × 2= Birr 8
3	Birr 4×3 = Birr 12
4	Birr 4×4= Birr 16
5	Birr 4×5 = Birr 20

In the above table, notice that the amount earned per bottle of soft drink is constant, Birr 4, but the number of bottles varies. You can use a **variable** to represent the number of bottles of soft drink sold. The expression for the amount earned is Birr 4 × $\square$ or Birr 4 × n, where n is a variable. This expression can also be written as 4n, which means 4 times the value of n.

The expression 4n is called an **algebraic expression** because it contains variables, numbers, and at least one operation.

Definition 2.1: An algebraic expression is a mathematical expression which consists of variables and /or numbers, often with operation signs and grouping symbols.

Example 2

a+36, 8.x, 3c-d, 12 ÷ y (y ≠ 0), $\frac{9}{5}$, and 4h (e+f) are examples of algebraic expressions.

Algebraic expressions such as 3x, $\frac{y}{7}$, 4ab, $3a^2$ are called **terms.**

Definition 2.2: A term is an indicated product and may have any number of factors.

Recall that a fraction bar is a division symbol: $\frac{9}{5}$, or 9/5, means 9÷5.

Similarly, multiplication can be written in several ways. For example, "7 times x" can be written as 7.x, 7×x, 7(x) or simply 7x.

Example 3

$4\ (5)$	means	4×5
$10x$	means	$10 \times x$
ab	means	$a \times b$
$4.9a$	means	$4\times9\times a$
$7ab^2$	means	$7\times a\times b\times b$
$a[b(cd)]$	means	$a \times [b\times (c\times d)]$
$\frac{e}{8f}$	means	$e \div (8 \times f)$
$a\left(\frac{b}{4}\right)$	means	$a \times (b\div4)$

Notice that an algebraic expression consists of one or more terms:

a) each term is separated from another by addition or subtraction symbol. For example,

(i) $2x + 7y \longrightarrow (2x) \quad + \quad (7y)$

first term second term

(ii) $x + 2y + 3Z \longrightarrow (x) \quad + \quad (2y) + \quad (3z)$

first term second term third term

b) each term consists of a variable or a constant. For example,

(i) in $2x + 7y \longrightarrow$ 2, 7 are constants and x, y are variables.

(ii) in $x + 2y + 3z \longrightarrow$ 1, 2, and 3 are constants and x, y, and z are variables.

c) each term is a combination of product and quotient. For example, in $2x + \frac{9y}{4}$

(i) the first term is a product of 2 and x.

(ii) the second term is a combination of product and quotient.

According to the number of terms algebraic expressions are classified as **monomials, binomials**, etc.

1. When an algebraic expression contains a single term, it is called **monomial**.

   4, 4x, $5y^2$, abc, are some examples of monomials.

2. When an algebraic expression consists of two terms, it is called **binomial**.

   2x+3, x+ 5y, xy - 6, x^2y-3y are some examples of binomials.

Example 4

Identify the algebraic expression and classify them as monomial and binomial: (a) x + 2y (b) 3xy

Solution: a) x+ 2y contains 2 terms x and 2y. So it is a binomial.

b) 3xy contains only one term. So it is a monomial.

You can act as a translator in Mathematics, interpreting words and ideas and translating them into mathematical expressions. Study the following example.

Example 5

Mathematical statements	Algebraic Expressions
Three less than a number	$x - 3$
A number increased by 10	$y + 10$
One third of a number	$\frac{a}{3}$
Twice a number	$2c$
The sum of two numbers	$e + f$
Twice a number decreased by five	$2d - 5$
The quotient of a number and 8	$\frac{n}{8}$

Example 6

Study the following chart which shows common phrases that usually indicate the four operations.

Operations	Phrases	Mathematical statements	Algebraic expressions
Addition (+)	added to sum of plus more than increased by	4 added to a number The sum of a number and 30 81 plus some number Birr 7 more than the amount made yesterday Bekele's original guess, increased by 15	$w + 4$ $n + 30$ $81 + x$ $a + 7$ $y + 15$
Subtraction (-)	subtracted from difference of minus less than decreased by	5 subtracted from a number The difference of two scores A team of size S, minus 2 injured players 23 less than the club scored Almaz's test score, decreased by 2	$w -5$ $a – b$ $S – 2$ $c – 23$ $t – 2$
Multiplication (x)	multiplied by product of times twice of half of	The number of students, multiplied by 8 The product of two numbers 10 times your weight Twice your age half of Ayele's salary	$8. n$ $c. d$ $10.w$ $2.a$ $\frac{1}{2}s$
Division (÷)	divided by quotient of divided into ratio of per	A number divided by 4 The quotient of a number and 5 The number of desks divided in to 3 class rooms The ratio of 80 to the cost of the book The speed of the car is 60 km per hour	$n \div 4$ $a \div 5$ $d \div 3$ $\frac{80}{b}$ $\frac{60}{h}$ Km

Group work 2.1

Write each phrase as an algebraic expression.

a) The quotient of a number and 20.
b) A number decreased by 10.
c) 20 times the difference of x and 2.
d) 7 plus the product of a number and 8.

Note: The order in which we subtract and divide **affects** the answer.

Example 7

a) Four less than Abebe's height, in centimeter' can be translated in to algebraic expression as h-4. But answering as 4-h is incorrect.

b) Amira's daily expense, divided by eight can be translated in to algebraic expression as $\frac{e}{8}$ or e ÷8. But answering as $\frac{8}{e}$ or 8÷e is incorrect.

Let us study the use of algebraic expressions in everyday life in the following example.

Example 8

If a pen costs Birr x and a pencil costs Birr y, how much will 10 pens and 12 pencils cost?

Solution: 10 pens cost Birr 10x and
12 pencils cost Birr 12 y

Therefore 10 pens and 12 pencils, altogether, cost 10x + 12y.

How much will 20 pens and 30 pencils cost? Did you answer 20x + 30y?

Exercise 2.A

1. What are the different ways of verbally expressing the operation of addition?
2. Identify the algebraic expressions and classify them as a monomial or binomial.

a) $3x+5y$

b) y^2

c) $\frac{xy}{3}$

d) x^2+y^2

e) $5xy-1$

3. Match the mathematical statement in column A with its algebraic expression in column B.

Column A	**Column B**
i. The sum of a number and 6	a. $y-10$
ii. Ten subtracted from a number	b. $x-6$
iii. Seventeen divided by some number	c. $\frac{10}{n}$
iv. The ratio of a number to 10	d. $a+6$
v. The difference between 16 and a number	e. $\frac{e}{3}$
vi. Ten times a number	f. $10.r$
vii. The product of 6 and a number	g. $17 \div q$
viii. One third of a number	h. $6m$
ix. The quotient of 10 and a number	i. $\frac{p}{10}$
x. A number decreased by 6	j. $16-d$
	k. $\frac{100}{x}$
	l. $17y$
	m. $6 \div a$

4. Write an algebraic expression for each of the following mathematical statements.
   - a. 5 more than Kebede's age.
   - b. The product of 40 and a
   - c. 36 divided by b.
   - d. 14 less than c.
   - e. 60 increased by d.
   - f. 24 times Jemila's weight
   - g. P decreased by q
   - h. r divided by t
   - i. The quotient of two numbers
   - j. n subtracted from m
   - k. twice m plus 8
   - l. one quarter of some number
   - m. one third of the sum of two numbers

5. Write a mathematical statement for each of the following algebraic expressions.
   - a. x-12
   - b. $\frac{1}{4}.r$
   - c. y + 28
   - d. r+s
   - e. 100w
   - f. 100÷z
   - g. 6-t
   - h. 2(a-b)
   - i. $\frac{u+v}{10}$

6. If a book costs Birr x and a calculator costs Birr y, how much will 10 such books and 12 calculators cost?
7. Asfaw reads P pages each day of a 300 page book. Write an algebraic expression for how many days it will take Asfaw to read the book.
8. To rent a certain car for a day costs Birr 200 plus Birr 0.50 for every kilometer the car is driven. Write an algebraic expression to show how it costs to rent the car for a day.

2.1.2 The Value of Simple Algebraic Expressions

Activity 2.3

Simplify 1. $(4^2 + 4) \div (2^2 - 2)$

2. $24 + 8 \times 12 \div 4 - 2$

3. $16 \div 8 + 9 \times 7$

4. $8 - [14 \div (2 + 5)]$

In a term like '2x', 2 and x are called the **factors** of the term. 2 is called the **coefficient** of the variable x.

Example 9

a) The coefficient of a, in the term 7a, is 7.

b) In the algebraic expression 8x+3y, 8 is the coefficient of x and 3 is the coefficient of y.

Definition 2.3: Terms having all of their literal factors (or variables) the same are called like terms. Terms which have only some or none of their literal factors (or variables) as common factors are called unlike terms.

Example 10

a) 2a and 3a are like terms.

b) 4x and $\frac{x}{2}$ are like terms.

c) 3x and 5y are unlike terms.

Example 11

Identify the like terms

$3a, 2b, 7c, 5b, \frac{a}{3}, \frac{c}{4}, 10a$

Solution

i) 3a, $\frac{a}{3}$ and 10a are like terms.

ii) 2b and 5b are another like terms.

iii) 7c and $\frac{c}{4}$ are third group of like terms.

To **evaluate** an algebraic expression, you substitute a number for each variable in the expression. This replaces each variable with a number. Then calculate the result.

Example 12

Evaluate each expression for the given values.

a) $x + y$ for $x = 12$ and $y = 38$

b) $8ab$ for $a = 2$ and $y = 3$

c) $3a-b+15$ for $a = 10$ and $b = 3$

d) $5e + 6f$ for $e = 12$ and $f = 11$

e) $\frac{4m^2}{3n^2}$ for $m = 9$ and $n= 6$

Solution

a) Substitute 12 for x and 38 for y and carry out the addition:

$x + y = 12 + 38 = 50$

The number 50 is called the value of the expression.

b) Substitute 2 for a and 3 for b and multiply:

$8ab = 8.2.3=16.3 = 48$, 8ab means 8 times the product of a and b

a) $3a - b+15=3(10)-3+15$, Replace a with 10 and b with 3.

$= 30-3+15$, Do multiplication before addition anc

$= 42$ subtraction.

b) $5e + 6f = 5(12) + 6(11)$ Replace e with 12 and f with 11

$= 60 + 66$ multiplication

$= 126$ Addition

e) $\frac{4m^2}{3n^2} = \frac{4(9^2)}{3(6)^2}$ Replace m with 9 and n with 6

$= \frac{4(81)}{3(36)} = \frac{324}{108}$ Evaluate the numerator and the denominator separately.

$= 324 \div 108$ Then divide

$= 3$

Group work 2.2

Evaluate each expression for the given value of the variable.

a) $20b - 19$ for $b = 2$

b) $3a^2 - 5a$ for $a = 3$

c) $9 + 3x - 5y + 3$ for $x = 2$ and $y = 1$

d) $3m^3 + \frac{y}{5}$ for $m = 2$ and $y = 35$

Let us study operations on algebraic expressions:

Addition can be performed only between two or more like terms. (Why?)

Let us consider a very simple example. If you add 4 pencils and 3 pencils, altogether they are 7 pencils but 4 pencils and 3 pens added together will give 4 pencils + 3 pens. Similarly, in adding 4x and 3x, you will get 7x but adding 4x and 3y will give only 4x + 3y.

Rules of Addition:

In adding algebraic expressions,

i) You add like terms.

ii) While adding like terms only the numerical coefficients are added.

iii) Symbolically, addition of ax and bx is given by $ax + bx = (a+b)\,x$.

iv) In case of unlike terms, it will remain same, can not be simplified further.

The following example will illustrate the method of addition.

Example 13

Add:

a) $8x, 3x, 5x$

b) $2ab, 4ab, 7ab$

c) $4y, 7x, 2y, 3x$

d) $10x^2, 5y^2, 3x^2, 4y^2$

e) $6c, 4d$

Solution: a) $8x + 3x + 5x = (8+3+5)x = 16x$

b) $2ab + 4ab + 7ab = (2+4+7)ab = 13ab$

c) $4y + 7x + 2y + 3x = (4y+2y) + (7x + 3x)$ .... like terms are separated

$= (4 + 2)y + (7+3)x$

$= 6y + 10x$

d) $10x^2 + 5y^2 + 3x^2 + 4y^2 = (10x^2 + 3x^2) + (5y^2 + 4y^2)$ . . . like terms are separated

$= (10+3)x^2 + (5+4)y^2$

$= 13x^2 + 9y^2$

e) $6c + 4d = 6c + 4d$. This is what happens when the monomials are unlike terms

In case of subtraction also, you subtract a term from a like term.

Rules of subtraction:

While you do the subtraction of algebraic expressions,

i) subtract a term from a like term

ii) find the difference between their numerical coefficients

iii) symbolically, subtraction of bx from ax is given by $ax-bx = (a-b)x$. For example $7x-3x= (7-3)x= 4x$

iv) you cannot simplify, while you subtract a term from its unlike term.

The following example will illustrate the method of subtraction.

Example 14

Subtract

a) 4x from 9x b) 7y from 13y c) 10c from 17c

Solution: a) $9x-4x = (9-4)x = 5x$

b) $13y - 7y = (13-7)y = 6y$

c) $17c - 10c = (17-10)c = 7c$

Note: To simplify an algebraic expression containing like and unlike terms, the following steps are to be followed:

i) Group the like terms

ii) Find the sum or difference of the coefficients of the like terms in each group.

The following example will illustrate the method:

Example 15

Simplify

a) $8c + 5b + 9 + 3c - 2b - 7$

b) $15x + 9y - 3x + 4y + 6x - y + 1$

Solution: a) $8c + 5b + 9 + 3c - 2b - 7$

$= (8c + 3c) + (5b - 2b) + (9 - 7)$

$= (8+3)\,c + (5-2)b + 2$

$= 11c + 3b + 2$

b) $15x + 9y - 3x + 4y + 6x - y + 1$

$= (15x - 3x + 6x) + (9y + 4y - y) + 1$

$= (15-3+6)\,x + (9+4-1)\,y + 1$

$= 18x + 12y + 1$

Exercise 2.B

1. Evaluate

a) $4x$, for $x = 3$

b) $\frac{x+y}{9}$ for $x = 12$ and $y = 6$

c) $8x-1$, for $x = 2$

d) $\frac{r-t}{8}$ for $r = 14$ and $t = 6$

e) $\frac{2u+3v}{6}$, for $u = 3$, and $v = 2$

f) $\frac{r}{t}$, for $r = 16$ and $t = 2$

g) $\frac{x+y}{7}$, for x= 15 and y=20

i) $\frac{m^2-n^2}{3}$, for m=6 and n=3

h) $\frac{9m}{q}$, for m = 6 and q = 18

2. Identify the like terms

a) 3x, 2y, x

c) 2z, 8z, 3y, 5y, z

b) 7u, $3u^2$, 5u, $4u^2$

3. If x = 6, y= 3 and z = 2, find the value of

a) $x \div y + xy$

c) $xy \div z - yz$

e) $\frac{x+y+z}{11}$

b) $x^2 + y^2 + z^2$

d) $x^2 - xy + z$

4. Add the monomials

a) 2x, 3x, 6x, x

c) 3xy, 7xy, 5xy

b) $2y^2$, $7y^2$, $9y^2$

d) 5b, 5b, 3b, 8b

5. Perform the indicated operations

a) 2x + 3y + 4z + 5x + 8y- 2z

c) $d^2 + e^2 + 4f^2 + 3d^2 + 2e^2 - 3f^2$

b) 4e + f + 3h +e – 2h+ 2f

6. Subtract a) 2x from 10x

c) 20z from 31z

b) 3y from 15 y

7. Simplify a) 4x + y + 6z – x +2y- 3z

b) 8r +2q +3t-7r –q -2t

c) 10t- 4t+ 8q+2r-3q+5r

2.2 Equations and Inequalities

Activity 2.4

Tell whether the given number makes the mathematical sentence true.

a) 15 = x + 7 ; 8

c) 18 - x = 1; 17

b) 14 +y = 19; 4

d) 3x = 21 ; 6

Equations

Do you know the difference between a mathematical expression and an equation?

A Mathematical **expression** is a number or a combination of numbers and literal numbers, using the signs of fundamental operations. Whereas an **equation** is equality of expressions. An equation has an equal sign; an expression does not have an equal sign.

Eleni has 22 books. This is 9 more than Kelemua has, this situation can be written as an equation.
An equation is like a balanced scale.

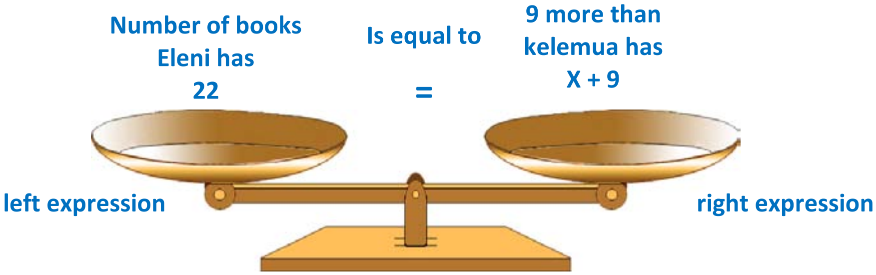


Figure 2.2

Just as the weights on both sides of a balanced scale are exactly the same, the expressions on both sides of an equation represent exactly the same value.

Activity 2.5

Identify each of the following as a Mathematical expression or an equation.

i) $2 + d$

ii) $3 + d = 5$

iii) $2x$

iv) $2x = 10$

v) $x + y + 3$

vi) $x - 2 = 5$

As a Mathematical statement of equality, equations show that two numbers or groups of numbers are equal. For example, $6 + 4 = 10$ shows the equality of expression. Equations also use variables that stand for numbers. You can use a variable even though you may not know what it represents. For example, a + 2 = 6. The variable a represents the number or **unknown** (4 in this example) for which we are solving.

Example 16

Let us consider the statement that 'when 7 is added to a number it gives 9' or ' add 7 to a number to get 9'. What is the number?

Solution: we can change the statement as' when 7 is added to x, it gives 9', i.e.

$x + 7 = 9$

Example 17

Consider the following statements:

a) A number added to 4 is equal to 13.
b) 5 subtracted from a number is equal to 24.
c) 3 times a number is 21.
d) A number divided by 7 gives 2.
e) Product of a number with itself is 36.

Now taking the unknown number on consideration as x, you can write the above statements as:

a) $x + 4 = 13$
b) $x - 5 = 24$
c) $3x = 21$
d) $\frac{x}{7} = 2$
e) $x^2 = 36$

The equation x + 4 = 13 contains a variable. The equation is neither true nor false until x is replaced with a number. You **solve** the equation when you replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution.** The solution to x+4 = 13 is 9 because 9 + 4 = 13. Can you solve x + 10 = 12 mentally?

Let us study the following example which deals with finding a value for the unknown which makes the given equation true by **substitution**.

Example 18

Which of the numbers 1, 2, 3 or 4 make x + 10 =12 true?

Solution: 1 + 10 = 12 implies 11 =12 . . . . False
(substituting x =1 in the equation x+ 10 =12)
2 + 10 = 12 implies 12 = 12 . . . . True
(substituting x =2 in the equation x + 10 = 12)
3 + 10 = 12 implies 13 = 12 . . . False
(substituting x =3 in the equation x + 10 = 12)
4 + 10 = 12 implies 14 = 12 . . . False
(substituting x=4 in the equation x+10 =12)

You can see that x=2 makes the equation x+10=12 true. That is, x =2 is a solution to the equation x+10 =12.

Example 19

Which of the numbers 6, 8 or 10 is the solution of 12r = 96?

Solution:

Replace r with 6	Replace r with 8	Replace r with 10
$12\ r = 96$	$12\ r = 96$	$12\ r = 96$
$12 \times 6 = 96$	$12 \times 8 = 96$	$12 \times 10 = 96$
$72 \neq 96$	$96 = 96$	$120 \neq 96$
this sentence is false	this sentence is true	this sentence is false

The solution is 8.

Group work 2.3

Find the solution of the following equations.

a) $x + 5 = 30$

b) $y - 30 = 40$

c) $10n = 90$

d) $\frac{m}{4} = 100$

Example 20

I think of a number and subtract 2 from it. My answer is 10. Which of the numbers 10, 12 or 13 I thought?

Solution: You can write the above statement as

$$x - 2 = 10$$

Replace x with 10	Replace x with 12	Replace x with 14
$x - 2 = 10$	$x - 2 = 10$	$x - 2 = 10$
$10 - 2 = 10$	$12 - 2 = 10$	$14 - 2 = 10$
$8 \neq 10$	$10 = 10$	$12 \neq 10$
This sentence is false	This sentence is true	This sentence is false

The number I thought of is 12.

Example 21

If it takes you 5 hours to travel 250 kilometers in a car, what is the average speed of the car? (use the equation $250 = 5r$, where r is the average speed of the car)

Solution: Solve $250 = 5r$ mentally (Ask yourself, what number multiplied by 5 equals 250?)

$250 = 5 \times 50$ You know $5 \times 50 = 250$

$250 = 250$

The solution is 50. Therefore, the value of r is 50.

Group work 2.4

The expression 60m gives the number of seconds in m minutes. Evaluate 60m for $m = 9$. How many seconds are there in 9 minutes?

Example 22

Michael sold his house for Birr 100,000. This price is four times the amount he originally paid for it 20 years ago. Which of the amounts 20,000, 25,000, or 30,000 did he originally pay for the house?

Solution: You may use the equation

$$4x = 100,000$$

where x represents the amount he originally paid for the house, and 100,000 represents the selling price.

Replace x with 20,000	Replace x with 25,000	Replace x with 30,000
$4x = 100,000$	$4x = 100,000$	$4x = 100,000$
$4 x 20,000 = 100,000$	$4 \times 25,000 = 100,000$	$4 \times 30,000 = 100,000$
$80,000 \neq 100,000$	$100,000 = 100,000$	$120,000 \neq 100,000$
This sentence is false	This sentence is true	This sentence is false

The amount he originally paid is Birr 25,000.

Exercise 2.C

1. Match the sentences in column A with its equation in column B.

Column A	**Column B**
i. Two more than a number equals twelve	a. $3a=18$
ii. Five less than a certain amount of Birr equals Birr ten	b. $\frac{p}{4}=10$
iii. Three times the age of a man equals eighteen	c. $n + 2 = 12$
iv. The quotient of the price of a book and Birr 4 equals 10	d. $b-5 =10$
	e. $4p = 10$
	f. $2n = 12$

2. Tell whether the equation is **true** or **false** using the given value of the variable.

   a. $k + 4 = 14$; $k = 16$ c. $10\ d = 300$; $d = 30$

   b. $p -8 = 17$; $p = 25$ d. $t \div 7 = 2$; $t = 2\ 1$

3. Name the number that is a solution of the given equation.

   a. $x + 15 = 19$; 4, 5, 6 c. $13t = 52$; 3, 4,5

   b) $y – 11= 18$; 29, 30, 31 d. $q \div 10=6$; 50, 60, 70

4. Write the equation for each of the following.

   a. A number plus four is nine.

   b. A number decreased by three is sixteen.

   c. The product of a number and six equals 48.

   d. The quotient of a number and three is 6.

5. Solve the following mentally.

   a. $x + 8 = 10$ b. $y -2 = 7$ c. $10m = 130$ d. $\frac{56}{n}=8$

6. Five is subtracted from a number. If the difference is seven, what was the original number?

7. The price of Almaz's sweater was reduced by Birr 30, write an algebraic expression if the sale price was Birr y.

8. If the cost of 5kg of sugar is Birr x, then what is the cost of 1kg of sugar?

Inequalities

Activity 2.6

Identify whether each of the following statements is True or False.

a) $4 (6 + 3) < 100$

b) $20 - 6 < 4 (3+2)$

c) $10 - 3 > 24 - 5 (3)$

d) $3 (10 - 3) \neq 4 (7 - 1)$

Do you know how to represent two expressions separated by an inequality sign? Two expressions separated by an inequality sign form an **inequality.** An inequality shows that the two expressions are **not** equal. Unlike the equations you have worked with, an inequality has many solutions.

An inequality uses one of the following symbols:

Symbol	Meaning	Word phrases
$<$	Is less than	Fewer than, below
$>$	Is greater than	More than, above
$\leq$	Is less than or equal to	At most, no more than
$\geq$	Is greater than or equal to	At least, no less than

Study the follow example

Example 23

Statements	Symbols
a) Twice a number is greater than 10	$2x > 10$
b) The quotient of a number and 3 is less than or equal to 2	$\frac{n}{3} \leq 2$
c) Ten decreased by a number is greater than or equal to seven	$10 - y \geq 7$
d) Eight times a number is less than sixteen	$8m < 16$

Exercise 2.D

1. Write an inequality for each of the following mathematical statements.
   a. A number minus two is greater or equal to ten.
   b. Three more than twice a number is less than twenty.
   c. Half of a number is less than or equal to six.
   d. Product of a number with itself is greater than hundred.
   e. A number divided by 3 is less or equal to ten.

2. Match the mathematical statement with its corresponding inequality from the column on the right.

Column A	**Column B**
i. The temperature today will be at most 24°c.	a. $y < 10$
	b. $n > 40$
ii. All numbers greater than 24	c. $t \leq 24$
iii. The price of a soft drink is below Birr 10	d. $m < 40$
	e. $x > 24$
iv. The family spent more than Birr 40 for dinner.	f. $p > 14$
	g. $c > 10$

3. Determine which of the given numbers are solutions of the given inequality.
   a. $x + 7 < 20$; 3, 5, 15, 20
   b. $a - 28 > 30$; 200, 100, 50, 30
   c. $\frac{y}{6} \leq 8$; 72, 54, 48, 6
   d. $8t \geq 96$; 5, 10, 14, 20
   e. $\frac{108}{x} \geq 36$; 2, 3, 4, 5

UNIT SUMMARY

Important facts you should know:

- A quantity which can take various numerical value is called **variable** and quantity which has a fixed numerical value is called **constant**.
- A number or combination of numbers and literal numbers, using the four operations (+, -, ×, ÷) is called **algebraic expression**.
- A **term** is an indicated product and may have any number of factors.
- (+) or (-) signs separate an algebraic expression into different parts. Each part is called a term of the expression.
- Terms in an algebraic expression which have the same literal factors are called **like terms**, otherwise they are **unlike terms**.
- Like terms can be added or subtracted together to make a single term.
- While doing the addition or subtraction of two or more like terms, only the numerical coefficients are added or subtracted.
- In adding or subtracting algebraic expressions, we collect different groups of like terms and find the sum or difference of like terms in each group.
- An expression which contains one term is called **monomial**, and which contains two terms is called **binomial**.
- An **equation** is equality of expressions.
- You solve an equation when you replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution**.
- Two expressions separated by an inequality sign form an **inequality**.

REVIEW EXERCISE

1. **Match the mathematical statement in column A with its appropriate algebraic expression or equation in column B.**

	Column A	**Column B**
i.	Two numbers that differ by 9	a. $x-10=9$
ii.	Two numbers with a sum of 7	b. $\frac{mn}{2}$
iii.	Three- fourths of a number	c. $2(m+n)$
iv.	The quotient of 120 and a number	d. $a+b=7$
v.	Two numbers such that one is 7 larger than the other	e. $ab-1=53$
vi.	Two numbers such that one number is 9 less than the other	f. $\frac{3}{4}q$
vii.	Half of the product of two numbers	g. $x-y=9$
viii.	Twice the sum of two numbers	h. $r=7+u$
ix.	Ten less than a number is nine	i. $\frac{120}{P}$
x.	One less than the product of two numbers is 53	j. $9-x=y$
		k. $\frac{4}{3}t$
		l. $\frac{1}{2}(a+b)$

2. **Evaluate**

   a) $\frac{x - y}{3}$ when x is twice y and x = 18

   b) $\frac{a + b}{4}$ when a is twice b and a = 16

   c) $\frac{x + y}{2}$ when y is twice x and x = 6

   d) $\frac{a - b}{3}$ when a is three times b and a = 18

3. Write algebraic expressions which represent the following mathematical statements
   a) If n + 3 is a whole number, what is the next whole number after it?
   b) If m + 2 is an odd number, what is the preceding odd number?
4. Write an algebraic expression for Rahel's age after 7 years, if she is 3 years older than Mulu and Mulu is a years old at present.
5. Identify whether each of the following statements is true or false?
   a) For any whole number x, the numbers x and x + 7 differ by 7.
   b) If Ahmed ran at x kilometers per hour for 3 hours, then he ran 3x kilometers.
   c) If Meseret ran at x kilometers per hour for 10 kilometers, she ran for $\frac{10}{x}$ hours.
   d) Three consecutive odd numbers can be represented by x, x+1 and x + 2.
   e) If a = 1, b = 2 and c = 3, then $\frac{a^2+2b+3c}{7}$ is equal to 2.

6. Classify each of the following as either an expression or an equation.
   a) The quotient of a number and 10 is 7.
   b) W increased by 20.
   c) The difference of 3 times a number and 7 is 2.
   d) Five plus 2 times a number is 13.

7. Which of the following given numbers can be in the solution of the inequality $2 + x > 7$?

   a) 4 b) 10 c) 5 d) 0

8. Beza spent Birr 2. She has Birr 5 left. How much money did she have before she spent Birr 2?

9. Fatuma had Birr 32 when she returned home from the supermarket. If she spent Birr 17 at the supermarket, did she have Birr 52 or Birr 49 before she went shopping?

10. Write an inequality for each situation.
    a) There are at least 28 days in a month.
    b) The temperature is above 30°c.
    c) There are no more than 350 people in the show room.
    d) Fewer than 100 people attended the meeting.

11. There are 120 eighth – graders at a school. If there are 30 more girls than boys, how many eighth- grade boys are there?

    a) 45 b) 55 c) 75 d) 95

12. Solve the equation $\frac{d}{15} = 8$.

UNIT 3

FRACTIONS, DECIMALS AND THE FOUR OPERATIONS

Unit Outcomes: After completing this unit you should be able to:

- know types of fractions.
- understand concept of percentage and principles of conversion of percentage to fraction and decimal.
- know the method of comparing fractions.
- perform the four basic operations on fractions and decimals.

Introduction

In earlier grades, you have learnt about fractions. After a review of your knowledge about fractions, you will continue studying fractions, decimals and the four operations in the present unit. Here, you will learn about types of fractions, conversion of percentages to fractions and decimals, comparing fractions and performing the four basic operations on fractions and decimals.

3.1 Types of Fractions

Activity 3.1

1. Find a fraction which is represented by the diagram shown below.

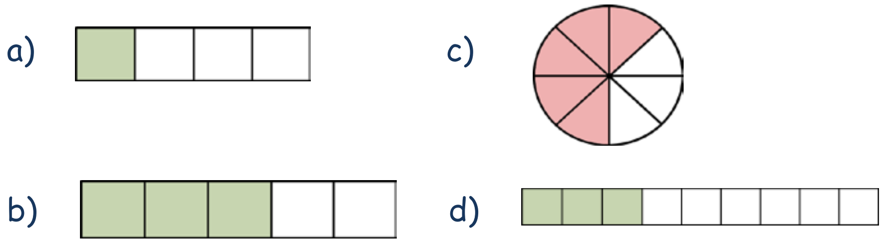


Figure 3.1

2. Write each fraction in simplest form.

a) $\frac{18}{20}$ b) $\frac{42}{60}$ c) $\frac{24}{40}$ d) $\frac{42}{56}$ e) $\frac{65}{75}$

3. Use <, >, or = to Compare each pair of fractions.

a) $\frac{7}{8} \square \frac{3}{8}$ d) $\frac{10}{15} \square \frac{8}{15}$

b) $\frac{4}{5} \square \frac{6}{10}$ e) $\frac{4}{9} \square \frac{7}{9}$

c) $\frac{6}{12} \square \frac{4}{8}$ f) $\frac{7}{16} \square 1$

Do you remember what you have studied about fractions in you grade 4 mathematics lessons? In this sub-unit you will study about types of fractions.

Remember that a **fraction** is a number (usually written as $\frac{a}{b}$, where a and b are whole numbers and b is not 0) equal to the quotient of a and b or a divided by b. Fractions are used in everyday life. For example, you can find what fraction of a week 4 days is:

4 days= $\frac{4}{7}$ week or

you can find what fraction of a month 7 days (a week) is:

$$7 \text{ days} = \frac{7}{30} \text{month.}$$

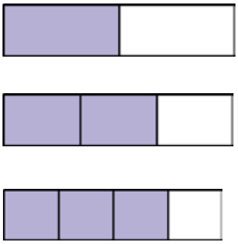
For the fraction $\frac{3}{4}$, the number 3 is called the **numerator** and the number 4 is called **denominator**

$$\begin{array}{l} 3 \longleftarrow \textbf{numerator} \\ \overline{4} \longleftarrow \textbf{denominator} \end{array}$$

The denominator of a fraction tells us the number of equal parts into which a whole has been divided and the numerator tells us how many of these parts are being considered. Thus $\frac{3}{4}$ tells us that the whole (a cup) has been divided in to 4 equal parts and that 3 parts are being used.

Figure 3.2

You can represent fractions by using a diagram such as the following.



1 part shaded out of 2. Thus $\frac{1}{2}$ is shaded.

2 parts shaded out of 3. Thus $\frac{2}{3}$ is shaded.

3 parts shaded out of 4. Thus $\frac{3}{4}$ is shaded.

Figure 3.3

Observe, infractions such as $\frac{1}{2}$ or $\frac{2}{3}$ or $\frac{3}{4}$, that the value of the numerator is less than the value of the denominator. Such fractions are called **proper fractions.**

A **proper fraction** has a value less than one; its numerator is smaller than its denominator.

Example 1

$\frac{1}{2}, \frac{1}{3}, \frac{1}{10}, \frac{1}{12}, \frac{4}{7}, \frac{3}{8}$ **are some examples of proper fractions.**

Can you give an example of a proper fraction of your own?

How much sleep do you get at night? Doctors recommend that we get 8 to $8\frac{1}{2}$ hours of sleep. What fraction is equivalent to $3\frac{1}{4}$?

Numbers such as $8\frac{1}{2}$ and $3\frac{1}{4}$ are called **mixed numbers.**

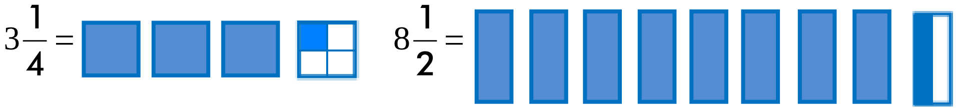


Figure 3.4

Mixed numbers show the sum of a whole number and a fraction. Mixed numbers can also be written as fractions.

Activity 3.2

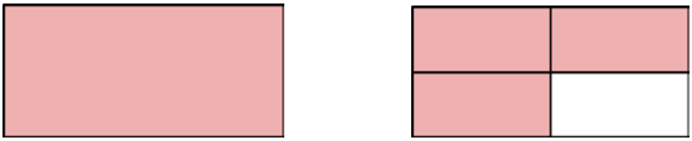
Work with a partner.

Materials: paper, pencil, ruler

Draw a **model for** $1\frac{3}{4}$

- Draw a rectangle like the one shown below. Shade the rectangle to represent 1.

- Draw an identical rectangle beside the first one. Separate the rectangle on the right into four equal parts to show fourths. Shade three parts to present $\frac{3}{4}$



- Separate the whole number portion in to one-fourths.

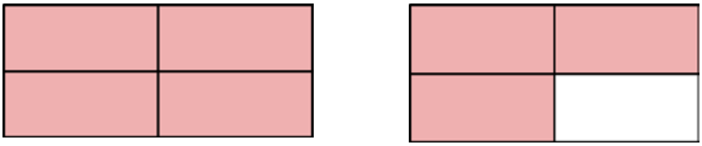


Figure 3.5

- How many shaded one-fourths are there?
- What fraction is equivalent to $1\frac{3}{4}$?

A fraction, like $\frac{8}{5}$ or $\frac{5}{4}$ with a numerator that is greater than or equal to the denominator is called an **improper fraction.**

From the Activity, you can conclude that it is possible to express a mixed number as an improper fraction. Here is one such example.

Example 2

Write $3\frac{1}{2}$ as an improper fraction.

Solution: Find the number of parts in the whole numbers. Then add the fraction.

$$\frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} = \frac{7}{2}$$

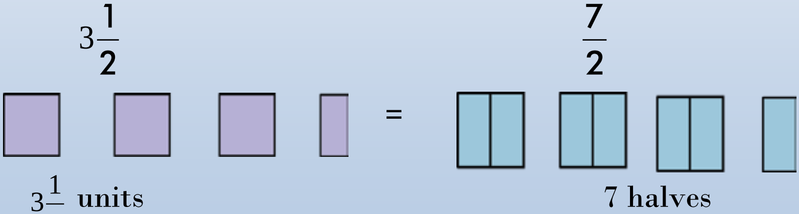


Figure 3.6

A short-cut is to multiply the whole number by the denominator and add the numerator. Then write this sum over the denominator.

$$\overset{+}{} \; 3\frac{1}{2} = \frac{(3\times 2)+1}{2} = \frac{7}{2} \; \underset{\times}{}$$

Study how you can write $3\frac{1}{4}$ as an improper fraction.

$$3\frac{1}{4} = 3 + \frac{1}{4} = \frac{13}{4}$$

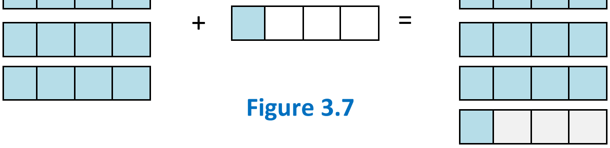
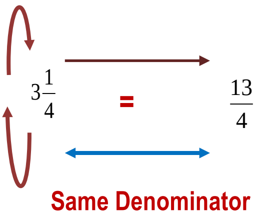


Figure 3.7



1. Multiply the denominator (4) by the whole number part (3)
2. Add the numerator (1). This is the new numerator.
3. Use the same denominator.

Here is the procedure. To write a mixed number as an improper fraction:

Converting mixed numbers to improper fractions

Step 1. **Multiply the denominator of the fraction by the whole number.**

Step 2. **Add the product from step 1 to the numerator of the old fraction.**

Step 3. **Place the total from step 2 over the denominator of the old fraction to get the improper fraction.**

Group work 3.1

Express as improper fraction.

a) $3\frac{5}{7}$ b) $6\frac{1}{4}$ c) $8\frac{1}{2}$

Example 3

Express each mixed number as improper fraction.

a) $4\frac{1}{2}$ b) $7\frac{2}{5}$

Solution

Find $4\frac{1}{2} = \frac{(4\times 2)+1}{2}$

$= \frac{9}{2}$

$7\frac{2}{5} = \frac{(7\times 5)+2}{5}$

$= \frac{37}{5}$

A whole number can be changed in to an improper fraction.

$$2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4}$$

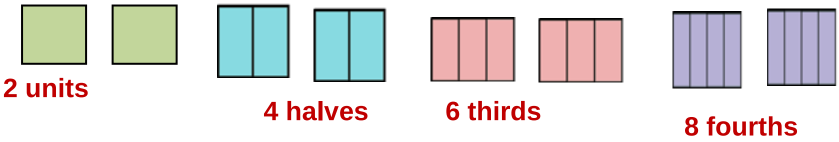


Figure 3.8

An improper fraction can also be changed in to either a whole number or a mixed number.

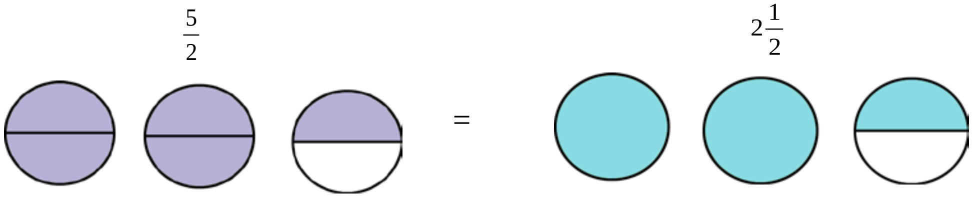
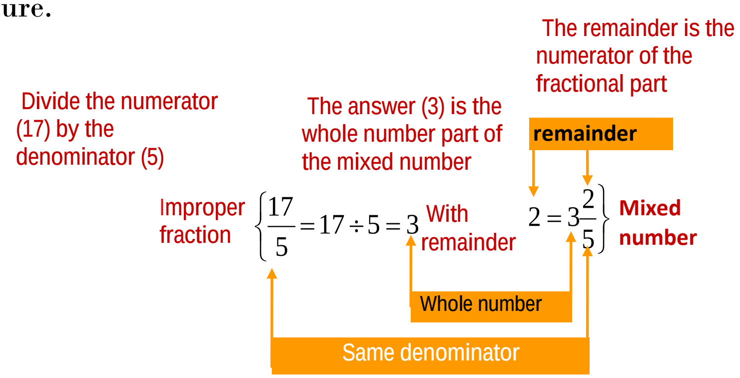


Figure 3.9

To convert an improper fraction to a whole number or a mixed number, divide the numerator by the denominator. Here is a diagram illustrating the procedure.



Note here the diagram for $\frac{17}{5}$

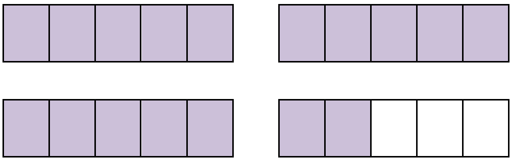


Figure 3.10

Note that $3\frac{2}{5} = 3 + \frac{2}{5}$, the sum of a whole number and a proper fraction. Similarly, $\frac{5}{3} = 1\frac{2}{3}$ and $\frac{8}{5} = 1\frac{3}{5}$.

Converting improper fractions to whole or mixed numbers:

Step 1. Divide the numerator of the improper fraction by the denominator.

Step 2. a) If you have no remainder, the quotient is a whole number.

b) If you have a remainder, the whole number part of the mixed number is the quotient. The remainder is placed over the old denominator as the proper fraction of the mixed number.

Example 4

Convert each improper fraction to a mixed number in simplest form or a whole number.

a) $\frac{21}{4}$ b) $\frac{24}{3}$ c) $\frac{77}{8}$

Solution: a) $\frac{21}{4} = 5\frac{1}{4}$ since $\begin{array}{r} 5 \\ 4\overline{)21} \\ \underline{20} \\ 1 \end{array}$

b) $\frac{24}{3} = 8$ since $\begin{array}{r} 8 \\ 3\overline{)24} \\ \underline{24} \\ 0 \end{array}$

c) $\frac{77}{8} = 9\frac{5}{8}$ since $\begin{array}{r} 9 \\ 8\overline{)77} \\ \underline{72} \\ 5 \end{array}$

Exercise 3.A

1. Identify whether each of the following statements is true or false.

a) $\frac{n}{n} = 1$ for any number n different from zero.

b) $\frac{n}{1} = n$ for any number n.

c) $\frac{0}{n} = 0$ for any number n different from zero.

d) $\frac{15}{16}$ is an improper fraction.

e) $\frac{n}{0}$ is not defined for any number n different from 0.

f) $\frac{47}{5} = 9\frac{2}{5}$

g) $\frac{23}{6} = 5\frac{1}{6}$

2. Classify the given fraction as proper or improper.

a) $\frac{13}{15}$ b) $\frac{17}{5}$ c) $\frac{9}{9}$ d) $\frac{0}{5}$ e) $\frac{8}{1}$

3. Write the fraction as a mixed number.

a) $\frac{21}{10}$ c) $\frac{18}{7}$ e) $\frac{29}{6}$ g) $\frac{69}{9}$ i) $\frac{101}{10}$

b) $\frac{46}{5}$ d) $\frac{59}{8}$ f) $\frac{39}{2}$ h) $\frac{97}{3}$ j) $\frac{98}{9}$

4. Write the mixed number as an improper fraction.

a) $8\frac{1}{7}$ c) $6\frac{1}{10}$ e) $1\frac{2}{11}$ g) $8\frac{3}{10}$ i) $2\frac{1}{16}$

b) $7\frac{1}{9}$ d) $5\frac{3}{11}$ f) $4\frac{2}{13}$ h) $9\frac{4}{11}$ j) $9\frac{7}{8}$

5. A person slept for 7 hours. What fraction of the day (24 hours) is that?

6. A woman has worked for 5 hours. If her work day is 8 hours long, what fraction of the day has she worked?

7. What fraction of an hour (60 minutes) is fourty-five minutes?

8. A cake was cut in to 8 equal parts. Five pieces were eaten.

   a) What fraction of the cake was eaten?

   b) What fraction of the cake was left?

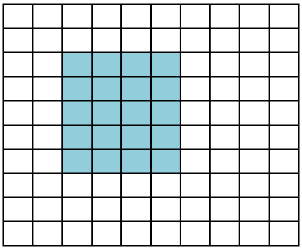
3.2. Percentage as Fractions

Activity 3.3

Work with a partner.

Materials: grid paper, markers

- Draw three 10 ×10 squares on your grid paper.
- For each percent below, shade three different 10 × 10 grids, each in a different way.



a) 60% b) 25% c) $35\frac{1}{2}\%$

- How can you find the percent represented by the shaded area at the right if you don't count the squares:

In this sub-unit you will deal with expressing a percentage as a fraction. The shaded area in the grid at the right shows that 43 out of 100 are shaded. Another name for the fraction $\frac{43}{100}$ is 43 **percent.**

A **percent** is a quotient that compares a number to 100. In symbols: $\frac{n}{100} = n\%$

The symbol % means **percent** or **per hundred** or **for every hundred.**

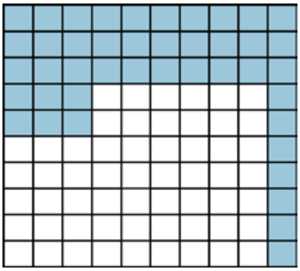


Figure 3.12

Example 5

Express each fraction as a percentage.

a) $\frac{37}{100} = 37\%$

b) a student answered 43 out of 100 = 43%

c) $9\frac{1}{2}$ **per hundred** $= 9\frac{1}{2}\%$

Example 6

Write a percent to represent the number of shaded squares.

Solution: The grid has one hundred squares in all. Count the number that are shaded.

There are 41 squares shaded. So, 41% represents the shaded area.

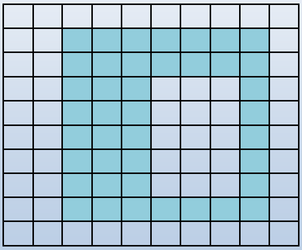


Figure 3.13

From the above discussion, perhaps you have got some idea about percentage. Now, we write 3% = 3 parts out of 100 equal parts $= \frac{3}{100}$

So, here we get a relation between **percentage** and **fraction.** Similarly, we can write 1% = 1 per hundred $= \frac{1}{100}$

Therefore 20% $= \frac{20}{100} = \frac{1}{5}$, 25% $= \frac{25}{100} = \frac{1}{4}$, 60% $= \frac{60}{100} = \frac{3}{5}$, etc

Conversion of fraction into percentage:

Step 1. **Multiply both numerator and denominator by 100.**

Step 2. **Convert $\frac{1}{100}$ to '%' symbol.**

Step 3. **Simplify the fractional part if required**

Group work 3.2

Express as percentage

a) 0.28   b) $\frac{3}{80}$ c) 0.7   d) 3.6

Example 7

Express each fraction as a percentage.

a) $\frac{4}{5}$ **b)** $\frac{3}{8}$ **c)** $\frac{6}{17}$ **d) 0.4**   **e) 2.5**

Solution: **a)** $\frac{4}{5} = \frac{4\times100}{5\times100}$ .... **Step 1**

$= \left(\frac{4\times100}{5}\right)\times\frac{1}{100}$ .... **Step 2**

$= \left(\frac{4\times100}{5}\right)\%$ .... **Step 3**

= 80%

Therefore, $\frac{4}{5}$ **=80%**

Another way to express a fraction as a percentage is to find an equivalent fraction with denominator of 100.

$$\frac{4}{5}=\frac{4\times 20}{5\times 20}=\frac{80}{100}=80\%$$

b) $\frac{3}{8}=\frac{3\times 100}{8\times 100}=\left(\frac{3\times 100}{8}\right)\times\frac{1}{100}=37.5\%$

c) $\frac{6}{17}=\frac{6\times 100}{17\times 100}=\left(\frac{6\times 100}{17}\right)\times\frac{1}{100}=35\frac{5}{17}\%$

d) 0.4 = $\frac{4}{10}=\frac{4\times 100}{10\times 100}=\left(\frac{4\times 100}{10}\right)\times\frac{1}{100}=40\%$

Activity 3.4

Work with a partner.

Materials: paper and pencil

What percentage of the students in your class do you think are in each category? Estimate by using one of the choices listed at the right.

a) left handed
b) right handed
c) male
d) female
e) less than 4 years old
f) greater than 10 years old

0%
Less than 10%
About 25%
About 50%
At least 75%
100%

To Write a percentage as a fraction, write a fraction with a denominator of 100. Then write the fraction in simplest form.

Example 8

Express each percentage as a fraction and decimal

a) 20% **b) 45%** **c) $18\frac{2}{3}\%$**

Solution: **a) 20%=** $$\frac{20}{100} \overset{\div 20}{\underset{\div 20}{=}} \frac{1}{5}$$

20% = 0.20 = 0.2

b) 45% = $$\frac{45}{100} \overset{\div 5}{\underset{\div 5}{=}} \frac{9}{20}$$

45% = 0.45

c) $$\mathbf{18}\frac{2}{3}\% = \frac{18\frac{2}{3}}{100} = 18\frac{2}{3} \div 100 = \frac{56}{3} \div 100$$

$$= \frac{56}{3} \times \frac{1}{100}$$ **To divide by 100, multiply by** $\frac{1}{100}$.

$$= \frac{56}{300} \overset{\div 4}{\underset{\div 4}{=}} \frac{14}{75}$$

$$\mathbf{18}\frac{2}{3}\% = \frac{18.666\ldots}{100} = \frac{18.\dot{6}}{100} = 0.18\dot{6}$$

You have observed that in order to write a percentage as a decimal, you need to divide by 100 and remove the % symbol.

Exercise 3.B

1. Express each fraction as a percentage

a) $\frac{14}{15}$ c) $\frac{18}{25}$ e) $\frac{1}{8}$ g) $\frac{7}{7}$ i) $\frac{12}{25}$

b) $\frac{23}{30}$ d) $\frac{13}{20}$ f) $\frac{5}{8}$ h) $\frac{19}{20}$ j) $\frac{3}{50}$

2. Express each percentage as a fraction in simplest form and decimal

a) 55% c) 75% e) 90% g) $19\frac{1}{2}\%$ i) $9\frac{1}{4}\%$

b) 12% d) 10% f) $36\frac{2}{3}\%$ h) $14\frac{1}{3}\%$ j) $16\frac{1}{5}\%$

3. Express each decimal as a percentage

a) 0.18 c) 0.7 e) 0.375 g) 0.086

b) 0.01 d) 0.025 f) 0.681 h) 0.0625

4. Write a percentage to represent the shaded area.

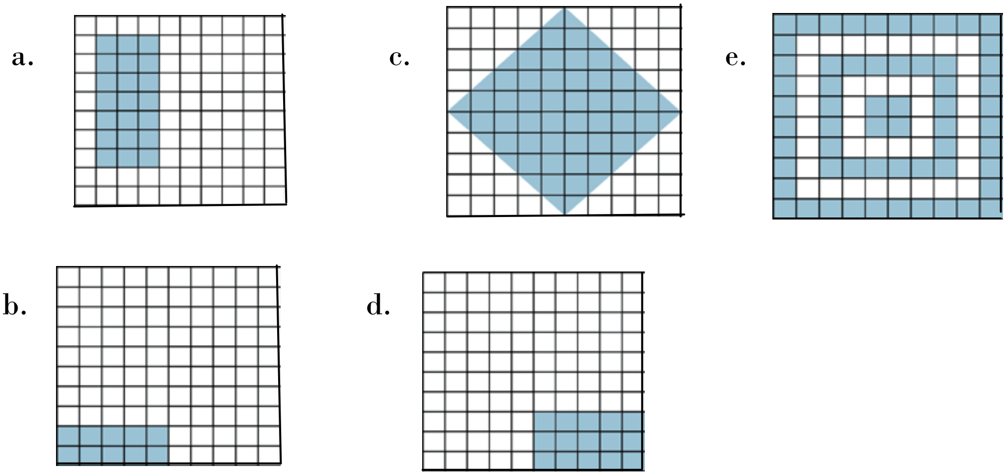


Figure 3.14

3.3. Comparison and Ordering of Fractions

Activity 3.5

Work with a partner.

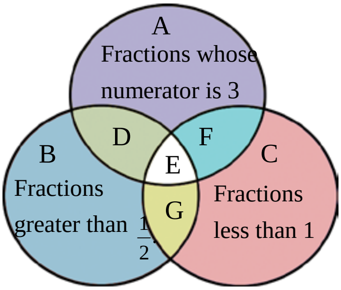


Figure 3.15

The figure below is called a **Venn diagram.** The circle at the lower left contains all fractions greater than $\frac{1}{2}$. The circle at the lower right contains all fractions less than 1. Thus the region labeled E or G, where only these two circles overlap, contains all fractions that are greater than $\frac{1}{2}$ and less than 1.

- Identify the region where each of the following fractions would be located.

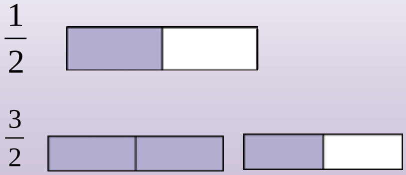
  $\frac{1}{8}, \frac{2}{3}, \frac{7}{4}, \frac{8}{9}, 1\frac{1}{2}, \frac{3}{5}, \frac{4}{4}, \frac{3}{7}$

The fact that the numerator and denominator of a fraction can be multiplied by the same non zero number without changing its value is used to compare fractions. In this sub-unit you will study comparison and ordering of fractions in more detail.

Consider the fractions $\frac{3}{2}$ and $\frac{1}{2}$ (two fractions with the same denominator). Which one do you think is greater? Here is the rule to compare two fractions:

Comparing fractions: Same denominator

To compare two fractions with the same denominator, compare the numerators. The one with the greater numerator is greater.



you can see that

$\frac{3}{2}$ is greater than $\frac{1}{2}$

Figure 3.16

Since $\frac{3}{2}$ has a greater numerator, $\frac{3}{2}$ is greater than $\frac{1}{2}$. In this case, we write $\frac{3}{2} > \frac{1}{2}$.

Can you tell which fraction is greater, $\frac{1}{2}$ or $\frac{3}{4}$?

Since $\frac{1}{2}$ and $\frac{3}{4}$ do not have the same denominator, we first must write both fractions as fractions with the same denominator. Here is the way to do it:

Comparing fractions: Different denominators

To compare two fractions with different denominators, write both fractions with a denominator equal to the product of the original ones.

$$\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

$$\frac{1}{2} = \frac{4}{8}$$

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

$$\frac{3}{4} = \frac{6}{8}$$

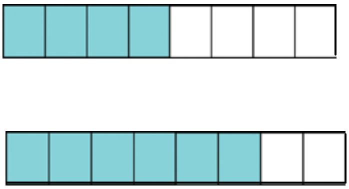


Figure 3.17

Since the numerator in $\frac{6}{8}$ (6) is greater than the one in $\frac{4}{8}$ (4), $\frac{3}{4}=\frac{6}{8}$ is greater than $\frac{4}{8}=\frac{1}{2}$.

In this case, we write $\frac{6}{8}>\frac{4}{8}$ or $\frac{3}{4}>\frac{1}{2.}$

A generalization of comparing fractions is given as follows. Study the example given below.

Example 9

In basic science class, Lemlem has earned 30 points out of possible 35 points on tests. In English class she worked hard writing short story and presentation, earning 42 out of a possible 48 points. In which class has Lemlem earned a great portion of the possible points.

Solution: First write each fraction in simplest form.

$$\frac{30}{35}\overset{\div 5}{\underset{\div 5}{=}}\frac{6}{7} \qquad \frac{42}{48}\overset{\div 6}{\underset{\div 6}{=}}\frac{7}{8}$$

To compare $\frac{6}{7}$ and $\frac{7}{8}$, rewrite each fraction using the same denominator. Then you need only compare the numerators.

$$\frac{6}{7}=\frac{6\times 8}{7\times 8}=\frac{48}{56} \qquad \frac{7}{8}=\frac{7\times 7}{8\times 7}=\frac{49}{56}$$

Now, compare $\frac{49}{56}$ and $\frac{48}{56}$. Since 49>48, then $\frac{49}{56} > \frac{48}{56}$., and Lemlem has earned a greater portion of the possible points in English than in basic science.

Note: 1. If two fractions have the same denominator, then fraction which has greater numerator is greater. Thus $\frac{4}{7} > \frac{2}{7}$ and $\frac{11}{20} > \frac{9}{20}$.

2. If the numerator of two fractions are equal, the fraction which has smaller denominator is greater. Thus $\frac{5}{9} > \frac{5}{11}$.

Group work 3.3

Which one is the least?

$\frac{3}{5}$, $\frac{4}{7}$ or $\frac{5}{8}$?

Example 10

Compare the fractions $\frac{9}{25}$ and $\frac{13}{40}$.

Solution: $\frac{9}{25} = \frac{9}{25} \times \frac{40}{40} = \frac{360}{1000}$

And $\frac{13}{40} = \frac{13 \times 25}{40 \times 25} = \frac{325}{1000}$

Since 360 > 325

Then $\frac{360}{1000} > \frac{325}{1000}$

Thus, $\frac{9}{25} > \frac{13}{40}$

Example 11

Arrange

a) $\frac{3}{4}, \frac{4}{3}$ **and** $\frac{6}{7}$ **in an ascending order.**

b) $\frac{3}{8}, \frac{1}{2}$ **and** $\frac{7}{5}$ **in a descending order.**

Solution

a) $\frac{3}{4} = \frac{3\times3\times7}{4\times3\times7} = \frac{63}{84}$

$\frac{4}{3} = \frac{4\times4\times7}{3\times4\times7} = \frac{112}{84}$

And $\frac{6}{7} = \frac{6\times4\times3}{7\times4\times3} = \frac{72}{84}$

Since 112>72>63, then

$\frac{112}{84} > \frac{72}{84} > \frac{63}{84}$

Therefore, $\frac{4}{3} > \frac{6}{7} > \frac{3}{4}$

That is, $\frac{3}{4}, \frac{6}{7}, \frac{4}{3}$ **are in ascending order.**

b) $\frac{3}{8} = \frac{3\times2\times5}{8\times2\times5} = \frac{30}{80}$

$\frac{1}{2} = \frac{1\times8\times5}{2\times8\times5} = \frac{40}{80}$

And $\frac{7}{5} = \frac{7\times2\times8}{5\times2\times8} = \frac{112}{80}$

Since 112 > 40 > 30, then

$\frac{112}{80} > \frac{40}{80} > \frac{30}{80}$

Thus, $\frac{7}{5} > \frac{1}{2} > \frac{3}{8}$

That is $\frac{7}{5}, \frac{1}{2}, \frac{3}{8}$ **are in descending order.**

Group work 3.4

Which one is greater?

$\frac{5}{6}$ or $\frac{7}{8}$

Example 12

Robel walks $\frac{3}{5}$ part of a certain distance and Molla walks $\frac{5}{8}$ part of the same distance in the same time. Who walks faster?

Solution.

$$\frac{3}{5}=\frac{3\times 8}{5\times 8}=\frac{24}{40} \quad \text{and} \quad \frac{5}{8}=\frac{5\times 5}{8\times 5}=\frac{25}{40}$$

Since 25>24, we see that $\frac{25}{40}>\frac{24}{40}$. That is, $\frac{5}{8}>\frac{3}{5}$

Therefore, Molla walks faster.

Exercise 3.C

1. Find the greater of the two numbers.

a) $\frac{5}{18},\frac{7}{18}$

b) $\frac{4}{11},\frac{5}{11}$

c) $\frac{3}{20},\frac{1}{20}$

d) $\frac{7}{12},\frac{9}{10}$

e) $\frac{7}{16},\frac{6}{15}$

f) $\frac{4}{7},\frac{14}{15}$

g) $\frac{7}{6},\frac{9}{10}$

h) $\frac{5}{14},\frac{3}{28}$

i) $\frac{5}{8},\frac{11}{10}$

j) $1\frac{4}{7},1\frac{5}{7}$

k) $6\frac{1}{3},6\frac{2}{5}$

l) $11\frac{2}{7},11\frac{3}{8}$

2. Arrange the fractions in ascending order.

a) $\frac{5}{2}, \frac{4}{3}, \frac{7}{4}$

b) $\frac{8}{15}, \frac{14}{35}, \frac{11}{21}$

c) $\frac{5}{6}, \frac{1}{18}, \frac{23}{36}$

d) $\frac{3}{7}, \frac{4}{9}, \frac{15}{21}$

e) $\frac{2}{30}, \frac{1}{10}, \frac{3}{5}$

3. Arrange the fractions in descending order.

a) $\frac{2}{3}, \frac{5}{6}, \frac{3}{8}$

b) $\frac{7}{2}, \frac{3}{4}, \frac{5}{16}$

c) $\frac{9}{10}, \frac{7}{6}, \frac{11}{15}$

d) $\frac{4}{5}, \frac{5}{6}, \frac{7}{12}$

e) $\frac{3}{7}, \frac{5}{14}, \frac{8}{21}$

4. Senait reads 24 out of 84 pages of a book within a day. But Hanan reads 21 out of 63 pages of another book within a day, who reads faster?

3.4 Operations on Fractions

3.4.1 Addition and Subtraction of Fractions

Activity 3.6

Add or subtract. Write each answer in simplest form.

a) $\frac{2}{7} + \frac{3}{7}$

b) $\frac{6}{13} - \frac{2}{13}$

c) $\frac{5}{9} - \frac{5}{9}$

d) $\frac{8}{12} + \frac{7}{12}$

e) $\frac{8}{9} + \frac{5}{9}$

f) $\frac{6}{15} - \frac{4}{15}$

g) $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + ... + \frac{9}{10}$

Do you remember what you have leant about addition and subtraction of fractions with the same denominators in your previous mathematics lessons? To add (subtract) fractions with the same denominators, add (subtract) the numerators.

Suppose a man spends about $\frac{1}{3}$ of his weekly income on food, $\frac{1}{6}$ on clothes and $\frac{1}{9}$ on entertainment. What is the fraction of money spent per week on food and entertainment? To find the fraction, you must add $\frac{1}{3}$ and $\frac{1}{9}$.

Activity 3.7

Work with a partner

Materials: fraction models

- To add $\frac{1}{2}$ and $\frac{1}{4}$, the common unit of measure is fourths.

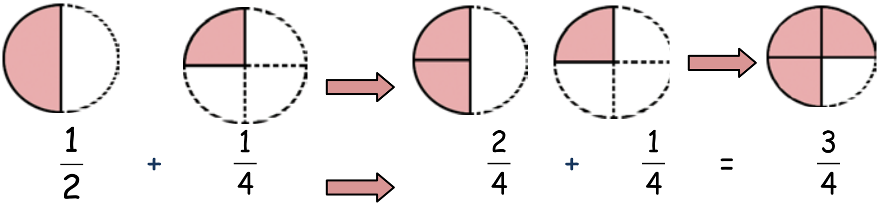


Figure 3.18

What conclusion can you draw about units of measures for fractions that are to be added or subtracted?

From the Activity you may conclude the following: To find the sum or difference of two fractions with different denominators, rename the fractions as fractions with the same denominator. Then add or subtract and simplify.

That is, if $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions (where b, d≠0), then

$$\text{(i)}\ \frac{a}{b}+\frac{c}{d}=\frac{a\times d}{b\times d}+\frac{c\times b}{d\times b}=\frac{ad+cb}{bd}$$

$$\text{(ii)}\ \frac{a}{b}-\frac{c}{d}=\frac{a\times d}{b\times d}-\frac{c\times b}{d\times b}=\frac{ad-cb}{bd}\ (ad-cb>0)$$

Now solve the problem at the beginning of this section .

$$\frac{1}{3}+\frac{1}{9}=\frac{3}{9}+\frac{1}{9}=\frac{4}{9}$$

The man spends about $\frac{4}{9}$ of his weekly income on food and entertainment.

Example 13

Add a) $\frac{1}{5}$ and $\frac{1}{2}$ **b)** $\frac{2}{5}$ and $\frac{1}{3}$ **c)** $2\frac{3}{10}+\frac{7}{20}+1\frac{3}{5}$

Solution. a) Here in $\frac{1}{5}$ and $\frac{1}{2}$, the denominators are 5 and 2.

You can write

$$\frac{1}{5}=\frac{1\times 2}{5\times 2}=\frac{2}{10}\ \text{and}\ \frac{1}{2}=\frac{1\times 5}{2\times 5}=\frac{5}{10}$$

Therefore $\frac{1}{5}+\frac{1}{2}=\frac{2}{10}+\frac{5}{10}=\frac{2+5}{10}=\frac{7}{10}$

b) Here, the denominators are 5 and 3.

Now $\frac{2}{5}=\frac{2\times 3}{5\times 3}=\frac{6}{15}$ and $\frac{1}{3}=\frac{1\times 5}{3\times 5}=\frac{5}{15}$

Therefore $\frac{2}{5}+\frac{1}{3}=\frac{6}{15}+\frac{5}{15}=\frac{6+5}{15}=\frac{11}{15}$

c) Observe that $2\frac{3}{10}=2+\frac{3}{10}$ and $1\frac{3}{5}=1+\frac{3}{5}$

Thus $2\frac{3}{10}+\frac{7}{20}+1\frac{3}{5}=2+\frac{3}{10}+\frac{7}{20}+1+\frac{3}{5}=2+1+\frac{3}{10}+\frac{7}{20}+\frac{3}{5}$

= $3+\frac{3\times 2}{10\times 2}+\frac{7}{20}+\frac{3\times 4}{5\times 4}$ **(why)?**

= $3+\frac{6}{20}+\frac{7}{20}+\frac{12}{20}$

$=3+\frac{6+7+12}{20}=3+\frac{25}{20}$

$=3+\frac{20+5}{20}$

$=3+\frac{20}{20}+\frac{5}{20}$

$=3+1+\frac{5}{20}$

$=4+\frac{5}{20}$

$=4\frac{1}{4}$

Therefore $2\frac{3}{10}+\frac{7}{20}+1\frac{3}{5}=4\frac{1}{4}$

Group work 3.5

Add

a) $3\frac{1}{2}+2\frac{3}{4}$ b) $11\frac{2}{3}+3\frac{1}{2}$ c) $24\frac{1}{16}+21\frac{3}{4}$

Example 14

Subtract (a) $\frac{3}{7}$ **from** $\frac{11}{21}$ **(b)** $\frac{5}{12}$ **from** $\frac{17}{24}$

Solution. a) $\frac{11}{21}-\frac{3}{7}=\frac{11}{21}-\frac{9}{21}$ $\left(\text{because } \frac{3}{7}=\frac{3\times3}{7\times3}=\frac{9}{21}\right)$

$$=\frac{11-9}{21}=\frac{2}{21}$$

Therefore, $\frac{11}{21}-\frac{3}{7}=\frac{2}{21}$

b) $\frac{17}{24}-\frac{5}{12}=\frac{17}{24}-\frac{10}{24}=\frac{17-10}{24}=\frac{7}{24}$ $\left(\text{because } \frac{5}{12}=\frac{5\times2}{12\times2}=\frac{10}{24}\right)$

Therefore, $\frac{7}{24}-\frac{5}{12}=\frac{7}{24}$

Group work 3.6

Evaluate

$$18\frac{11}{12}-\left(9\frac{1}{4}+6\frac{2}{3}\right)$$

Example 15

Find the simplified value of $\frac{3}{2}+\frac{2}{3}-\frac{1}{5}$

Solution: $\frac{3}{2}+\frac{2}{3}-\frac{1}{5}=\frac{45}{30}+\frac{20}{30}-\frac{6}{30}=\frac{45+20-6}{30}=\frac{59}{30}=1\frac{29}{30}$

Exercise 3.D

1. Add. Then write each sum in simplest form.

a) $\frac{2}{5}+\frac{3}{4}$

b) $\frac{5}{6}+\frac{3}{8}$

c) $\frac{4}{15}+\frac{2}{25}$

d) $\frac{5}{14}+\frac{8}{7}$

e) $2\frac{3}{4}+1\frac{7}{8}$

f) $1\frac{5}{16}+2\frac{3}{8}$

g) $4\frac{3}{10}+\frac{9}{20}$

h) $3\frac{2}{7}+2\frac{5}{14}$

i) $\frac{3}{5}+\frac{5}{7}+2\frac{1}{35}$

j) $4+3\frac{5}{18}+2\frac{1}{9}$

k) $1\frac{7}{10}+2\frac{1}{20}+3\frac{4}{40}$

2. Subtract. Then write each difference in simplest form.

a) $\frac{3}{4}-\frac{1}{8}$

b) $\frac{7}{5}-\frac{3}{10}$

c) $\frac{5}{12}-\frac{7}{36}$

d) $7\frac{1}{2}-5$

e) $\frac{11}{4}-2\frac{1}{3}$

f) $3\frac{1}{5}-\frac{3}{8}$

g) $4\frac{1}{6}-2\frac{1}{5}$

3. Find the simplified value.

a) $\frac{7}{4}+\frac{5}{6}-\frac{1}{12}$

b) $\frac{3}{4}+\frac{7}{2}-\frac{1}{8}$

c) $\frac{5}{3}+\frac{3}{4}-\frac{1}{2}$

d) $\frac{4}{15}+\frac{7}{9}-\frac{1}{3}$

e) $\frac{7}{12}+\frac{5}{6}-\frac{3}{4}$

f) $\frac{1}{4}+\frac{1}{7}-\frac{1}{28}$

4. Does $\frac{3}{4}+\frac{5}{8}-\frac{5}{6}=\frac{5}{8}+\frac{5}{6}-\frac{3}{4}$? Explain.

5. A bottle contains $1\frac{1}{2}$ litres of water. If $\frac{1}{4}$ litre of water is used up from the bottle, how much water is left in it?

6. What must be added to $\frac{3}{10}$ to get $\frac{1}{2}$?

7. What must be subtracted from $\frac{7}{12}$ to get $\frac{1}{4}$?

8. Mesfin cuts a rope of length 9 meters in to two pieces. If one piece is $4\frac{1}{6}$ metres long, what is the length of the other piece?

9. A father left $\frac{1}{4}$ of his money to his daughter, $\frac{1}{2}$ to his wife, and $\frac{1}{8}$ to his son. What fraction of the money remained?

3.4.2 Multiplication and Division of Fractions

a) Multiplication of Fractions

As in the multiplication of whole numbers, multiplication of fractions and mixed numbers represents repeated addition.

The picture below shows 3 cups, each containing $\frac{1}{4}$ cup of sugar. How much sugar do they contain altogether? To find the answer we must multiply 3 by $\frac{1}{4}$, that is, we must find $3\times\frac{1}{4}$.

Figure 3.19

We have 3 one-quarter cups of sugar, which make $\frac{3}{4}$ cup. Thus to find the answer, we multiply 3 by $\frac{1}{4}$, obtaining

$$3.\frac{1}{4}=\frac{3}{4}$$

We can show the idea pictorially like this:

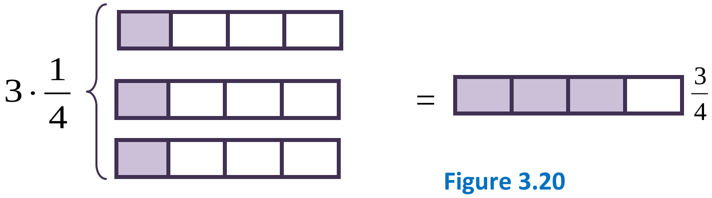
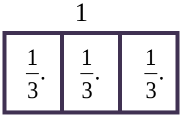


Figure 3.20

The diagram also suggests that multiplication is repeated addition; that is, $3\times\frac{1}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$. Similarly, if a recipe calls for $\frac{1}{3}$ cup of flour and we wish to make only $\frac{1}{2}$ of the recipe, we have to find $\frac{1}{2}$ *of* $\frac{1}{3}$ (which means $\frac{1}{2}\times\frac{1}{3}$ because "of" is translated as "times"), that is, $\frac{1}{2}\cdot\frac{1}{3}$. Here is a diagram to help you do it.



Each rectangle on the diagram represents $\frac{1}{3}$.

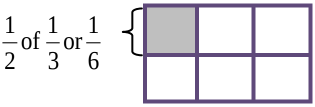


Figure 3.21

$$\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$$

Activity 3.8

Fill in the blanks

a) $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=3\times\square=\square$

b) $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=4\times\square=\square$

c) $\frac{3}{7}+\frac{3}{7}+\frac{3}{7}+\frac{3}{7}+\frac{3}{7}=\square\times\frac{3}{7}=\square$

Notice that you can also find the product of $\frac{1}{2}$ and $\frac{1}{3}$ as follows:

$$\frac{1}{2}\times\frac{1}{3}=\frac{1\times1}{2\times3}=\frac{1}{6}$$

Similarly $\frac{2}{9}\times\frac{4}{7}=\frac{2\times4}{9\times7}=\frac{8}{63}$

and $\frac{5}{3}\times\frac{9}{16}=\frac{5\times9}{3\times16}=\frac{45}{48}=\frac{15}{16}$ $(45\div3=15$ and $48\div3=16)$

Rule for Multiplying Fractions

The **product** of two fractions is a fraction whose numerator is the product of numerators of the given fractions and whose denominator is the product of their denominators.

In symbols, $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$

Example 16

Multiply. Write each product in simplest form.

a) $\frac{2}{3}\times\frac{5}{7}$ **b)** $\frac{2}{9}\times\frac{7}{2}$ **c)** $4\frac{2}{3}\times 9$

Solution:

a) $\frac{2}{3}\times\frac{5}{7}=\frac{2\times 5}{3\times 7}=\frac{10}{21}$

b) $\frac{2}{9}\times\frac{7}{2}=\frac{\overset{1}{\cancel{2}}\times 7}{9\times\cancel{2}}=\frac{7}{9}$

c) $4\frac{2}{3}=\frac{(4\times 3)+2}{3}=\frac{12+2}{3}=\frac{14}{3}$ or $4\frac{2}{3}=\frac{4}{1}+\frac{2}{3}=\frac{12+2}{3}=\frac{14}{3}$

Therefore, $4\frac{2}{3}\times 9=\frac{14}{\cancel{3}}\times\frac{\overset{3}{\cancel{9}}}{1}=42$

Note: Multiplying Mixed Numbers: To Multiply Mixed numbers, rename each mixed number as an improper fraction. Then multiply the fractions.

Example 17

Find $4\frac{1}{2}\times 1\frac{1}{3}$

Solution. $4\frac{1}{2}\times 1\frac{1}{3}=\frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}}=6$

b) Division of fractions

When you studied whole numbers in Unit 1, you saw how multiplication can be checked by division. The multiplication of fractions can also be checked by division, as you will see in this section on dividing proper fractions and mixed numbers.

Dividing proper fractions

The division of proper fractions introduces a new term the **reciprocal.** To use reciprocals, we must first recognize which fraction in the problem is the divisor. Let's assume the problem we are to solve $\frac{1}{4} \div \frac{2}{3}$. We read this problem as "$\frac{1}{4}$ divided by $\frac{2}{3}$." The divisor is the fraction after the division sign (or the second fraction). The steps that follow show how the divisor becomes a reciprocal.

Dividing proper fractions:

Step 1: Invert (turn up side down) the divisor. The inverted number is the reciprocal.

Step 2: Multiply the fractions.

Step 3: Reduce the answer to lowest terms.

Do you know why the inverted fraction number is a reciprocal? Reciprocals are two numbers that when multiplied give a product of 1. For example, 3 (which is the same as $\frac{3}{1}$) and $\frac{1}{3}$ are reciprocals because multiplying them gives 1.

Reciprocal: The product of a number and its reciprocal is 1.

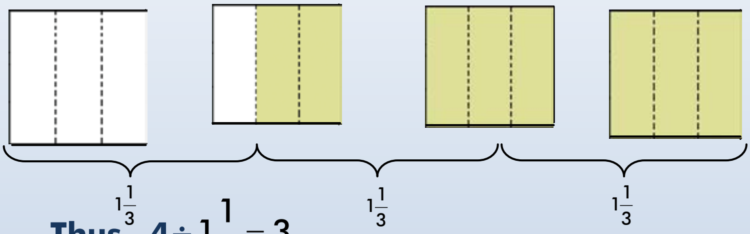
That is, for all fractions $\frac{a}{b}$, where a, b $\neq 0$, $\frac{a}{b} \times \frac{b}{a} = 1$.

Example 18

Suppose a girl figures that a person will drink $1\frac{1}{3}$ cups of orange juice for breakfast. And she buys 4 cups of orange juice for seven people. Will there be enough juice?

To solve this problem, we need to find how many $1\frac{1}{3}$ cups are in 4 cups. Divide 4 by $1\frac{1}{3}$.

Figure 3.22



Thus, $4 \div 1\frac{1}{3} = 3$

Figure 3.23

You can also divide by a fraction or mixed number. To do this multiply by its reciprocal.

$4 \div 1\frac{1}{3} = \frac{4}{1} \div \frac{4}{3}$ (Rename 4 as $\frac{4}{1}$ and $1\frac{1}{3}$ as $\frac{4}{3}$)

$= \frac{4}{1} \times \frac{3}{4}$ (Dividing by $\frac{4}{3}$ is the same as multiplying by $\frac{3}{4}$).

$= \frac{3}{1}$ or 3

4 Cups of orange juice will be enough for 3 people, not for 7 people.

Division of fractions and Mixed numbers: To divide by a fraction multiply by its reciprocal.

That is, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ where b, c and d $\neq 0$.

Thus $\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$

Can you change order in division as you do in multiplication?

Example 19

Is $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2} \div \frac{1}{4}$?

Now, $\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$ and $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$ but $\frac{1}{2} \neq 2$

So the statement is false. Therefore, we cannot change order in division. That is $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

Group work 3.7

Divide

a) $6 \div 4\frac{1}{2}$

b) $42 \div 4\frac{2}{3}$

c) $\frac{15}{16} \div 1\frac{1}{3}$

Now you are ready to divide mixed numbers by using improper fractions.

Dividing Mixed numbers

Step 1. Convert all mixed numbers to improper fractions.

Step 2. Invert the divisor (take its reciprocal) and multiply

If your answer is an improper fraction, reduce it to lowest terms.

Example 20

Divide

$6\frac{3}{4} \div 3\frac{5}{6}$

$= \frac{27}{4} \div \frac{23}{6}$ **step 1**

$= \frac{27}{4} \times \frac{6}{23}$ **step 2**

$= \frac{81}{46} = 1\frac{35}{46}$

Exercise 3.E

1. Multiply. Write each product in simplest form.

   a) $\frac{3}{5} \times \frac{10}{21}$
   d) $\frac{2}{7} \times \frac{21}{6}$
   g) $\frac{4}{5} \times \frac{2}{4} \times \frac{4}{6}$
   b) $\frac{5}{9} \times \frac{27}{35}$
   e) $\frac{9}{5} \times \frac{35}{36}$
   h) $6\frac{1}{8} \times \frac{8}{9}$
   c) $\frac{3}{4} \times \frac{8}{15}$
   f) $\frac{20}{3} \times \frac{9}{40}$
   i) $3\frac{1}{8} \times 3\frac{4}{5}$

2. Evaluate ab if a= $1\frac{5}{7}$ and b= $2\frac{5}{8}$.

3. Find the product $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots\ldots \times \frac{99}{100}$

4. Find the value of

   a) $\frac{1}{4}$ of 100
   c) $\frac{1}{2}$ of 64
   e) $\frac{7}{6}$ of 120
   b) $\frac{1}{7}$ of 98
   d) $\frac{3}{5}$ of 80

5. A book has 100 pages. Chala read $\frac{3}{10}$ of the book. How many pages are left to read?

6. Name the reciprocal of each number.

a) $\frac{3}{7}$ b) 4 c) $2\frac{4}{5}$ d) $4\frac{5}{6}$

7. Divide. Write each quotient in simplest form

a) $\frac{2}{5} \div \frac{1}{4}$ d) $8 \div 2\frac{1}{2}$ g) $1\frac{1}{9} \div 1\frac{2}{3}$

b) $\frac{3}{14} \div \frac{2}{7}$ e) $2\frac{1}{4} \div \frac{2}{3}$ h) $4\frac{1}{2} \div 6\frac{3}{4}$

c) $5 \div \frac{1}{6}$ f) $2\frac{2}{3} \div 5\frac{1}{3}$ i) $5\frac{1}{4} \div 3$

8. Will the quotient $5\frac{3}{8} \div 6\frac{3}{4}$ be a proper or a mixed number?

9. In a school with a total number of 2000 students, $\frac{3}{5}$ are girls. Find the number of boys.

3.5 Operations on Decimals

In this sub-unit you will deal with addition, subtraction, multiplication and division of decimals in more detail.

3.5.1 Addition and Subtraction of Decimals

The following Activity will help you revise what you have learnt about decimals in your previous mathematics lessons.

Activity 3.9

1. Fill in the blanks. The first one is done for you.

a) 3.42 = _3_ ones _4_ tenths _2_ hundredths.

b) 4.51 = ____ ones ___ tenths ___ hundredths.

c) 0.345 =____ tenths ___hundredths ___ thousandths.

d) 15.27 = ____ tens ___ ones ___ tenths ___hundredths.

2. Use >, < or = to compare the decimals

a) 0.3 __ 0.5
b) 0.04 __ 0.01
c) 1.31 __ 1.13
d) 5.08 ____ 5.8
e) 0.9____ 0.09
f) 0.7 _____ 0.71

3. Write 3.8, 3.79, 3.67 and 3.81 in order from least to greatest.

Do you remember how to add decimals? Adding decimals is like adding whole numbers. Make sure that you line up the decimal points before you add or subtract.

Adding Decimals

1. Line up the decimal points.
2. Write zeros so that both numbers have the same number of decimal places.
3. Add.

Example 21

Add 12.5 and 27.21

Solution. **First line up the decimal points**

$$\begin{array}{r} 12.50 \\ +\underline{27.21} \\ 39.71 \end{array}$$

You can write a zero so that (12.5=12.50) each addend has the same number of decimal places.

Therefore, 12.5+27.21=39.71

Example 22

A school paid Birr 234.50 for new jackets and Birr 175.35 for new shirts. What is the total cost?

Solution.

$$\begin{array}{r} 234.50 \\ +\underline{175.35} \\ 409.85 \end{array}$$

Therefore, total cost= Birr 409.85

Group work 3.8

Add

a)
$$\begin{array}{r} 382.41 \\ +\ 471.26 \\ \hline \\ \hline \end{array}$$

b)
$$\begin{array}{r} 766.62 \\ +865.33 \\ \hline \\ \hline \end{array}$$

Example 23

A bird model has a head 11.3 cm long and a neck 23.15cm. The rest of the body is 64.52cm long. How long is the entire bird?

Solution: **To answer, add.**

First line up the decimal points.

$$\begin{array}{r} 11.30 \\ 23.15 \\ +\underline{64.52} \\ \underline{98.97} \end{array}$$

(11.3 = 11.30)

The bird is 98.97cm long.

Figure 3.23

Example 24

Find the sum of 12.041, 26.706 and 321.24

Solution.

$$\begin{array}{r} 12.041 \\ +\ 26.706 \\ \underline{321.240} \\ \underline{359.987} \end{array} \qquad (321.24=321.240)$$

Group work 3.9

An elephant's speed is 40.001 kilometers per hour. A pig's speed is 17.601 kilometers per hour. What is the sum of the speeds of the elephant and pig?

Subtraction of decimal fractions can also be done in the same way as you did in case of whole numbers, only keep in mind the following steps:

Subtracting Decimals

1. **Line up the decimal points.**
2. **Write zeros so that both numbers have the same number of decimal places.**
3. **Subtract as with whole numbers.**

Example 25

Subtract 0.3 from 1.53

Solution.

$$\begin{array}{r} 1.53 \\ -\ \underline{0.30} \\ \underline{1.23} \end{array}$$

make sure to place the decimal point correctly

(0.3=0.30)

Example 26

Subtract 41.32 from 543.431

Solution: (41.32=41.320)

$$\begin{array}{r} 543.431 \\ -\ \underline{41.320} \\ \underline{502.111} \end{array}$$

Group work 3.10

Subtract

a)

$$\begin{array}{r} 3\ .\ 8\ 4 \\ -\ 1\ .\ 7\ 2 \\ \hline \\ \hline \end{array}$$

b) 27.51 from 347.82

Example 27

The weights of one bag of rice and one bag of wheat are 52.05kg and 63.375kg respectively. Which bag is heavier and by how much?

Solution

$$63.375>52.05$$

This implies that the bag containing wheat is heavier. And the difference is given as

$$\begin{array}{r} 63.375 \\ -\underline{52.050} \\ \underline{11.325} \end{array}$$

Thus, the bag containing wheat is heavier than the bag containing rice by 11.325kg.

Exercise 3.F

1. Add
   a) 3.21 and 4.015
   b) 0.04, 2.132 and 4.013
   c) 25.002, 40.115 and 13.101
   d) 10.134,9.021 and 120.412

2. Subtract
   a) 3.21 from 5.623
   b) 7.341 from 18.451
   c) 4.3 from 17.591
   d) 12.53 from 20.639

3. Last year 2.15 million people visited a park. This year 3.26 million visited. How many more people visited the park this year?
4. Abetu drove 215.355km from his house to his sister's house. His friend's house was 14.1 km shorter. How far did Abetu travel on his way to his friend's house?
5. An office building is 125.3m high. The building next to it is 40.45m higher than that. How high is the second building?
6. A rope is 80m long. Three pieces of length 13.25m, 21.4m, 18.3m are cut off. How much rope is left?

3.5.2 Multiplication of Decimals

Activity 3.10

Multiply

a) 35 × 21
b) 47 × 82
c) 124 × 35
d) 853 × 46
e) 236 × 103
f) 343 × 59

The multiplication of decimals is similar to the multiplication of whole numbers except for the additional step of placing the decimal in the answer (product). The product will have the same number of decimal places as the sum of the number of decimals in the factors.

Example 28

Multiply

a) 0.13 ← **Two decimal places**
$\times \underline{2}$
$\underline{0.26}$ ← **Two decimal places**

b) 1.4 ← **One decimal place**
$\times \underline{0.3}$ ← **One decimal place**
$\underline{0.42}$ ← **Two decimal places**

c) 2.37 ← **Two decimal places**
$\times \underline{0.8}$ ← **One decimal place**
$\underline{1.896}$ ← **Three decimal places**

What do you understand? The steps that follow simplify the procedure of multiplication of decimals.

Multiplying decimals

Step 1. Multiply the numbers as whole numbers ignoring the decimal points.

Step 2. Count and total the number of decimal places in the multiplier and multiplicand.

Step 3. Starting at the right in the product, count to the left the number of decimal places totaled in step 2. Place the decimal point so that the product has the same number of decimal places as totaled in step 2. If the total number of places is greater than the places in the product, insert zeros in front of the product.

Example 29

a) $6.3 \leftarrow$ **one decimal place**
$\underline{\times 1.2} \leftarrow$ **one decimal place**
126
$\underline{63}$
$7.56 \leftarrow$ **Two decimal places**

b) $2.13 \leftarrow$ **two decimal places**
$\underline{\times 3.5} \leftarrow$ **one decimal place**
1065
$\underline{639}$
$7.455 \leftarrow$ **Three decimal places**

Activity 3.11

Find the product in each case

a) 1.2 × 10
1.2 × 100
1.2 × 1000

b) 0.37 × 10
0.37 × 100
0.37 × 1000

c) 0.048 × 10
0.048 × 100
0.048 × 1000

d) 3.65 × 10
3.65 × 100
3.65 × 1000

The following example illustrates short cut steps to solve multiplication **problems involving multiples of 10 (10, 100, 1000, 10,000, etc).** Study the shift in decimal point.

Example 30

2.43×10 = 24.3 (1 decimal place to the right)
2.43×100= 243 (2 decimal places to the right)
2.43×1000=2430 (3 decimal places to the right)

What do you understand? You may follow the following steps to solve multiplication problems involving multiple of 10.

Step 1. **Count the zeros in the multiplier.**

Step 2. **Move the decimal point in the multiplicand the same number of places to the right as you have zeros in the multiplier.**

Exercise 3.G

1. Multiply

a) 0.12×3	d) 8.3×1.4	g) 0.47×0.32
b) 0.17×4	e)7.6×5.6	h) 1.23×4.8
c) 3.4×8	f) 4.25 ×2.3	i) 5.31×0.48

2. A piece of string is 0.32cm long. What is the total length of 12 such pieces of string?

3. The cost per hour to rent a medium-size car is Birr 36.75. What is the charge to rent this car for 9 hours?

4. Use >, < or = to compare the following

a) 1.5×1.2 ☐ 3.6×0.5

b) 3.2×2.4 ☐ 5.1×1.2

c) 0.34×1.3 ☐ 0.4×1.2

d) 7.75×1.5 ☐ 77.5×2.5

e) 0.86×0.8 ☐ 8.6×0.1

5. Alemu says that he runs about 1.35 km in each football game. How many kilometers does he run in 3.5 games (or in three and half games)?

3.5.3 Division of Decimals

Activity 3.12

What is the quotient when

a) 4 is divided by 0.5?

b) 2 is divided by 0.1?

If the divisor in your decimal division problem is a whole number, first place the decimal point in the dividend. Then divide as usual. If the divisor has a decimal point, complete the steps that follow.

Dividing Decimals

Step 1. **Make the divisor a whole number by moving the decimal point to the right.**

Step 2. **Move the decimal point in the dividend to the right the same number of places that you moved the decimal point in the divisor (step 1). If there are not enough places, add zeros to the right of the dividend.**

Step 3. **Divide as usual.**

Example 31

a) $8 \div 0.5 = \frac{8}{0.5} \times \frac{10}{10} = \frac{80}{5} = 16$

b) $27 \div 0.9 = \frac{27}{0.9} \times \frac{10}{10} = \frac{270}{9} = 30$

c) $0.36 \div 0.04 = \frac{0.36}{0.04} \times \frac{100}{100} = \frac{36}{4} = 9$

d) $\frac{15.6}{0.13} = \frac{15.6}{0.13} \times \frac{100}{100} = \frac{1560}{13} = 120$

The following example discusses **dividing decimals by powers of ten.** Study the shift in decimal point.

Example 32

a) 3.87÷10=.387 (1 decimal place to the left)

b) 3.87÷100=.0387 (2 decimal places to the left)

c) 3.87÷1000=.00387 (3 decimal places to the left)

d) 3.87÷10000=.000387 (4 decimal places to the left)

Activity 3.13

What is the quotient when 2.13 is divided by 10? by 100? by 1,000? by 10,000?

You may use the rule that follow:

Dividing decimals by powers of ten: To divide a decimal by 10, 100, 1000, etc. Shift the decimal point in the dividend to the left as many places as the number of zeros in the divisor.

Example 33

a) 0.4÷ 10=0.04
b) 12.6÷ 100=0.126
c) 34.5÷ 1,000=0.0345

Exercise 3.H

1. Divide

   a) 5÷0.1
   b) 80÷0.02
   c) 12÷0.06
   d) 12.8÷0.64
   e) 2.25÷1.5
   f) 3÷0.04
   g) 19.6÷0.14
   h) 25.6÷0.16
   i) 10÷0.001

2. Fill in the blank

   a) 4.27 ÷ 10 = ☐
   b) 4.27 ÷ ☐ = 0.427
   c) 4.27÷100 = ☐
   d) 4.27÷1000= ☐
   e) 0.56÷☐=0.056
   f) 5.6÷ ☐ =0.56
   g) 14.28÷ ☐ =0.1428

UNIT SUMMARY

Important facts you should know:

- **Types of fractions**
  - (i) **Proper:** value less than 1; numerator smaller than denominator.

    E.g. $\frac{3}{7}, \frac{7}{9}, \frac{8}{19}$
  - (ii) **Improper:** value equal to or greater than 1; numerator equal to or greater than denominator.

    E.g. $\frac{5}{5}, \frac{20}{13}$
  - (iii) **Mixed:** Sum of whole number greater than zero and a proper fraction.

    Eg. $6\frac{3}{4}, 7\frac{8}{9}$
- **Fractions conversions**
  - (i) Improper to whole or mixed: Divide numerator by denominator; place remainder over old denominator.

    Eg. $\frac{17}{4} = 4\frac{1}{4}$
  - (ii) Mixed to improper:

    $$\frac{\text{whole number} \times \text{Denominator} + \text{Numerator}}{\text{old denominator}}$$

    Eg. $4\frac{1}{8} = \frac{32+1}{8} = \frac{33}{8}$

- **Adding and Subtracting fractions**

  i) When denominators are the same, add numerators, place total over original denominator, and reduce to lowest terms.

  $$\frac{5}{9}+\frac{1}{9}=\frac{6}{9}=\frac{2}{3}$$

  ii) When denominators are different, change them to like fractions. Then add (or subtract) the numerators, place total over their common denominator, and reduce to lowest terms.

  Eg. $\frac{4}{5}+\frac{2}{7}=\frac{28}{35}+\frac{10}{35}=\frac{38}{35}=1\frac{3}{35}$

- **Adding and Subtracting Mixed numbers**

Convert the mixed numbers to improper fractions, then add (or subtract) by writing both fractions as equivalent ones with the same denominators, reduce to lowest terms.

Eg. $4\frac{2}{5}+2\frac{3}{4}=\frac{22}{5}+\frac{11}{4}=\frac{88}{20}+\frac{55}{20}=\frac{143}{20}=7\frac{3}{20}$

$$3\frac{1}{4}-1\frac{1}{8}=\frac{13}{4}-\frac{9}{8}=\frac{26}{8}-\frac{9}{8}=\frac{17}{8}=2\frac{1}{8}$$

- **Multiplying proper fractions**

  i) Multiply numerators and denominators

  ii) Reduce answer to lowest terms

  Eg. $\frac{3}{5}\times\frac{10}{18}=\frac{30}{90}=\frac{1}{3}$

- **Multiplying Mixed numbers**

  i) Convert mixed numbers to improper fractions.

  ii) Multiply numerators and denominators.

  iii) Reduce answer to lowest terms.

Eg. $1\frac{1}{8}\times2\frac{4}{5}=\frac{9}{8}\times\frac{14}{5}=\frac{126}{40}=\frac{63}{20}$

- **Dividing proper fractions**
  - **i) Invert divisor**
  - **ii) Multiply**
  - **iii) Reduce answer to lowest terms**

Eg. $\frac{2}{3}\div\frac{4}{9}=\frac{2}{3}\times\frac{9}{4}=\frac{18}{12}=\frac{3}{2}=1\frac{1}{2}$

- **Dividing Mixed numbers**
  - **i) Convert mixed numbers to improper fractions**
  - **ii) Invert divisor and multiply, If final answer is an improper fraction reduce to lowest terms.**

Eg. $1\frac{1}{2}\div1\frac{5}{8}=\frac{3}{2}\div\frac{13}{8}=\frac{3}{2}\times\frac{8}{13}=\frac{24}{26}=\frac{12}{13}$

- **Addition and Subtraction of decimals**
  - **i) Line up the decimal points**
  - **ii) Write zeros so that both numbers have the same number of decimal places**
  - **iii) Add or subtract as with whole numbers**

Eg.

$$\begin{array}{r} 13.40 \\ \underline{+5.12} \\ \underline{18.52} \end{array} \qquad \begin{array}{r} 24.963 \\ \underline{-\ \ 3.500} \\ 21.463 \end{array}$$

- **Multiplication of decimals**
  - **i) Multiply the numbers as whole numbers ignoring the decimal point.**

ii) Count and total the number of decimal places in the multiplier and multiplicand.

iii) Starting at the right in the product, count to the left the number of decimal places totaled in step 2. Place the decimal point so that the product has the same number of decimal places as totaled in step 2. If the total number of places is greater than the places in the product, insert zeros in front of the product.

Eg. 2.3 ⟵ one decimal place
× 0.6 ⟵ one decimal place
1.38 ⟵ two decimal places

- **Division of decimals**

i) Make the divisor a whole number by moving the decimal point to the right.

ii) Move the decimal point in the dividend to the right that you moved the decimal point in the divisor (step1).
If there are not enough places, add zeros to the right of the dividend.

iii) Divide as usual.

Eg. $$12 \div 0.25 = \frac{12}{0.25} \times \frac{100}{100}$$

$$= \frac{1200}{25} = 48$$

REVIEW EXERCISE

1. Match the fraction with its percentage.

A	B
a) $\frac{1}{8}$	i) 37.5%
b) $\frac{1}{6}$	ii) $33\frac{1}{3}\%$
c) $\frac{1}{3}$	iii) 50%
d) $\frac{3}{8}$	iv) 12.5%
e) $\frac{1}{2}$	v) $16\frac{2}{3}\%$
f) $\frac{3}{4}$	vi) $66\frac{2}{3}\%$
g) $\frac{2}{3}$	vii) 75%
	viii) 7.5%
	ix) 0.5%
	x) 5%

2. Find the value of the following.

a. 50% of 80

b. 35% of 60

c. $\frac{1}{4}$ of 100

d) $\frac{2}{5}$ of 120

e) $\frac{4}{3}$ of 450

3 Write each improper fraction as a mixed number or as a whole number.

a) $\frac{11}{2}$ = ________

b) $\frac{15}{8}$ = ________

c) $\frac{24}{7}$ = ________

d) $\frac{31}{6}$ = ________

e) $\frac{14}{3}$ = ________

f) $\frac{42}{14}$ = ____________

g) $\frac{13}{12}$ = ____________

h) $\frac{27}{8}$ = ____________

i) $\frac{47}{5}$ = ____________

4. Write each numbers as an improper fraction.

a) $2\frac{3}{4}$ = ________

b) $3\frac{5}{8}$ = ________

c) $7\frac{2}{5}$ = ________

d) $5\frac{2}{5}$ = ____________

e) $6\frac{1}{3}$ = ____________

f) $12\frac{3}{5}$ = ____________

5. Add or subtract.

a) $\frac{1}{8}+\frac{2}{3}$

b) $\frac{4}{9}+\frac{3}{4}$

c) $2\frac{1}{5}+1\frac{5}{6}$

d) $5\frac{3}{4}+2\frac{7}{8}$

e) $5\frac{1}{2}-2\frac{4}{5}$

f) $6\frac{1}{3}+1\frac{5}{8}$

g) $8-1\frac{2}{3}$

h) $4\frac{1}{6}-2\frac{1}{3}$

i) $3\frac{7}{8}-2\frac{3}{4}$

j) $18\frac{2}{5}-9\frac{1}{2}$

k) $8\frac{5}{8}-4\frac{3}{5}$

l) $17\frac{1}{3}+9\frac{4}{9}+2\frac{6}{7}$

6. Which fraction is equivalent to 957 $\frac{3}{5}$?

a) $\frac{4781}{5}$

c) $\frac{4783}{5}$

b) $\frac{4788}{5}$

d) $\frac{9573}{5}$

7. Multiply. Simplify to lowest terms

a) $2\frac{1}{3}\times 6\frac{2}{5}$

c) $1\frac{7}{8}\times\frac{5}{6}$

b) $9\times 2\frac{1}{2}$

8. Divide. Simplify to lowest terms

a) $\frac{5}{9}\div\frac{1}{2}$

c) $2\frac{1}{4}\div 1\frac{2}{3}$

b) $\frac{6}{11}\div\frac{5}{6}$

d) $5\frac{5}{6}\div 2\frac{2}{5}$

9. A bucket contains 20 $\frac{1}{2}$ litres of water. If 8 $\frac{1}{4}$ litres of water is used up, how much water remains in the bucket?

10. Nunu and her two friends ate lunch at a hotel. They decided to split the bill evenly. The total bill was Birr 82.50. How much was each person's share?

11. Melkamu measured the amount of rainfall at his house for 3 days. On Sunday, it rained 0.4 in. On Monday, it rained $\frac{5}{8}$ in. On Wednesday it rained 0.57 in. List the days in order from the least to the greatest amount of rainfall.

12. Find each product

a) $\begin{array}{r} 3.42 \\ \underline{\times\ 7.2} \end{array}$

b) $\begin{array}{r} 2.3 \\ \underline{\times\ 4.1} \end{array}$

c) $\begin{array}{r} 5.12 \\ \underline{\times\ 0.3} \end{array}$

d) $\begin{array}{r} 4.68 \\ \underline{\times\ 5.8} \end{array}$

e) 2.8× 0.05

f) 1.45× 0.7

13. Find each quotient

a) 4÷0.01 b) 0.3÷ 0. 03 c) 3.5÷0.7 d) 3÷0.003 e) 11÷0.001

14. A father gave away half of his property to his wife and the remaining was equally divided among his three children. If his total property was worth Birr 120,000, then find the share of each member of the family.

15. A pair of foot ball shoes weighs 1.213 kilograms. How much do 10 such pairs weigh? 100 pairs? 1000 pairs?

UNIT FOUR

DATA HANDLING

Unit Outcomes: After completing this unit, you should be able to:

- understand simple graphical representation of data
- know and calculate average of a given data

Introduction

You have some knowledge about data handling from your grade four mathematics. In this unit, you will deal with constructing bar graphs by collecting simple data from your lives. You will also deal with interpreting bar graphs and finding the average of numbers.

4.1. Further on Construction and Interpretation of Bar Graphs

Do you remember what you have studied in earlier grade about data handling? In this sub-unit you are going to study simple graphical representation of data.

The following Activities will help you get some idea on collecting data and drawing a graph to show your data.

Activity 4.1

Which is your favorite fruit?

Adugna made a list of fruits. He asked each student in the class to choose a favorite fruit from his list. He recorded the results of his survey in a table.

Fruit	Number of students
Banana	6
Pineapple	7
Mango	11
Orange	5
Papaya	8
Poam	3

Then the draw a picture graph.

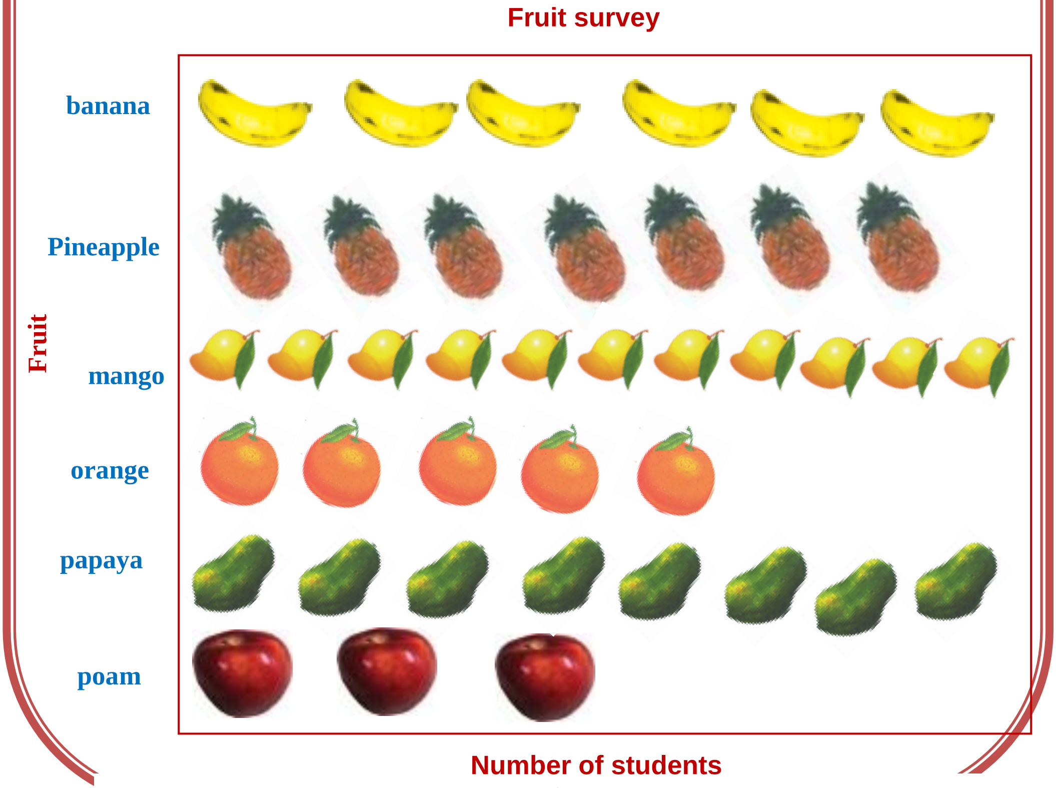


Figure 4.1

1. Which was the most liked of the fruits?
2. Which was the least liked fruit?
3. What is the total number of students surveyed?

- Carry out a survey of fruit with the students in your class. Collect your data in a table. Draw a picture graph to show your data.

Activity 4.2

For a class of 40 students a survey showed:

Born on	**Number**
Monday	5
Tuesday	7
Wednesday	6
Thursday	7
Friday	9
Saturday	2
Sunday	4
Total	40

A graph to show days on which students were born

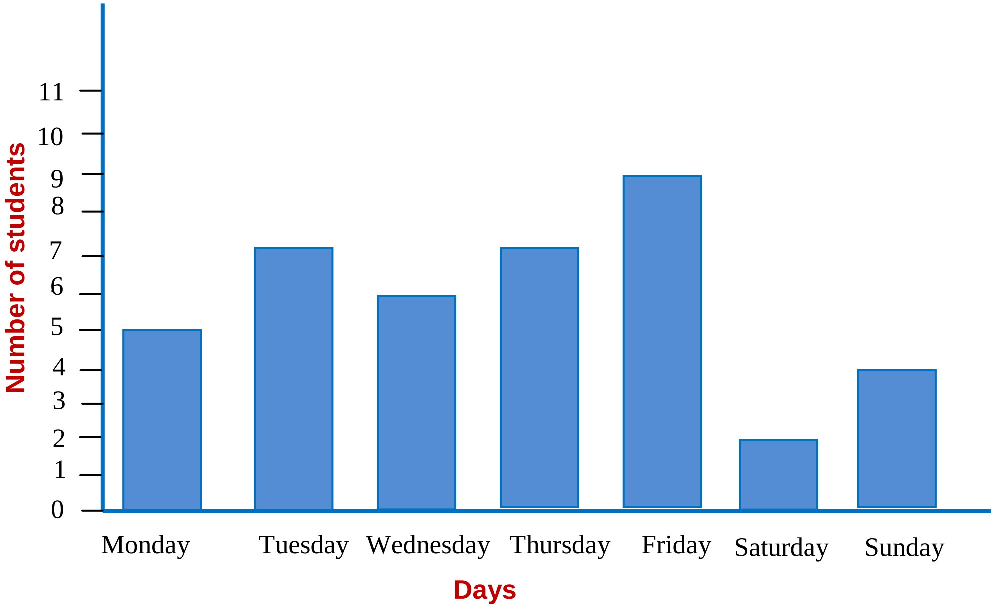


Figure 4.2

1. On which day were most students born?
2. On which day were very few students born?
3. On which day were the same number of students born?
    - Make a survey of birthdays in your class.
    - Draw a graph to show the results of your survey.

Activity 4.3

You will need a coin and some square cards all of the same size.

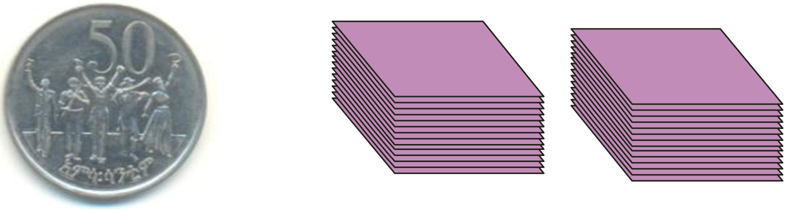


Figure 4.3

Toss the coin 10 times and record 'Head' or 'Tail' using the cards.

Your results might look like this:

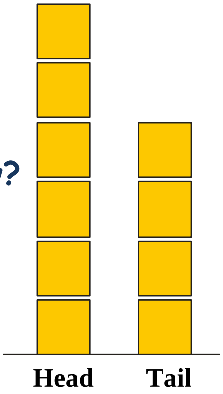


Figure 4.4

1. How many times did the 'Tail' show?
2. How many times did the 'Head' show?
3. Repeat the tossing of the coin 10 times. Did you get the same result?
4. Toss a coin 20 times. Record the 'Heads' or 'Tails' as shown below. This information is called data.

This is a useful way of recording data.

Head	8
Tail	12

The data can be shown on squared paper.

The number of 'Heads' and 'Tails' can easily be read by using the scale. So if the 'Heads' appear 8 times, you fill in 8 squares on the 'head' column.

The difference, if any, between the 'Head' and 'Tail' can be seen by comparing the heights of the coloured columns.

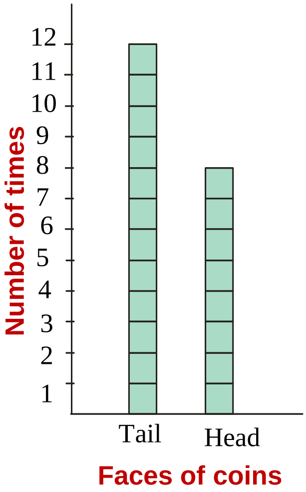


Figure 4.5

Data handling deals with collecting, organizing, and summarizing numerical facts. When the data is collected and displayed in a graph, you can look for trends and study details of the data.

A bar graph is a pictorial representation of numerical data by a number of bars of uniform width erected vertically (or horizontally) with equal spacing between the bars.

Bar graphs are used to compare numbers. The bar graph below shows the amount of money six children have. Bar graphs can be vertical or horizontal.

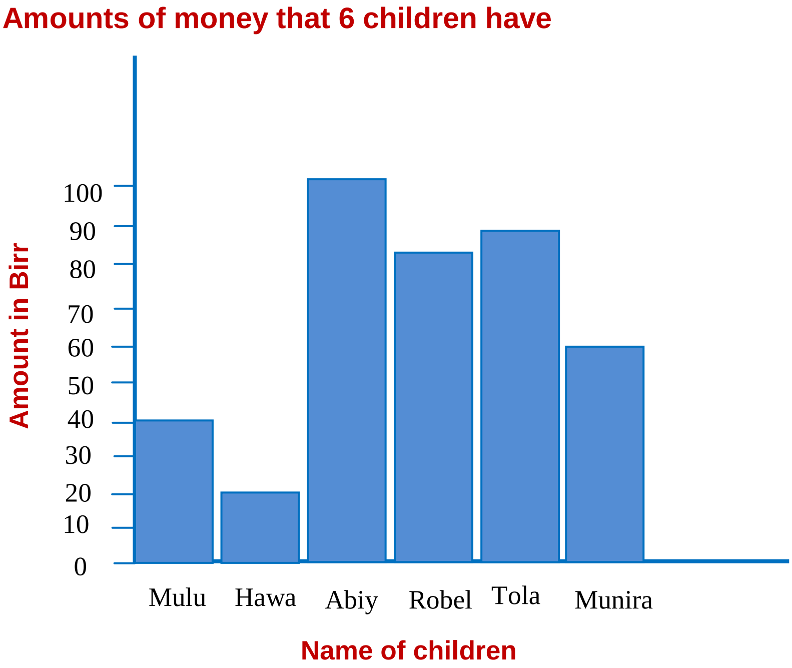


Figure 4.6

To read the graph:

- Find the bar marked Mulu on the horizontal axis
- Follow this bar to the end
- Look to the vertical axis. Read the number.

Mulu has Birr 40.

Use the bar graph to answer each of the following questions and check your answer with the solution given.

a) Who has most money?
b) Who has least money?
c) Has Tola less money than Munira?
d) Has Abiy more money than Robel?
e) How much money do the children have altogether?

Solution: a) Abiy b) Hawa c) No d) yes e) Birr 390

Example 1

The size of shoes worn by 30 students in a certain school are given below. Show the results of the survey on a bar graph. This information is called **raw data**. What conclusions can you draw from it?

34	37	36	37	36	34	37	38	35	37
37	34	35	37	34	36	34	36	38	34
38	37	36	37	34	35	38	37	36	37

Solution: First, we organize the information on a table.

Shoes sizes	Number of students
34	7
35	3
36	6
37	10
38	4
Total	30

Now you can draw a bar graph showing all the information on shoe sizes.

- Suitable titles for the graph would be size of shoes worn by 30 students.
- The horizontal axis is labeled 'shoe size'
- The vertical axis is labeled 'number of students'.
- What does each vertical square represent?

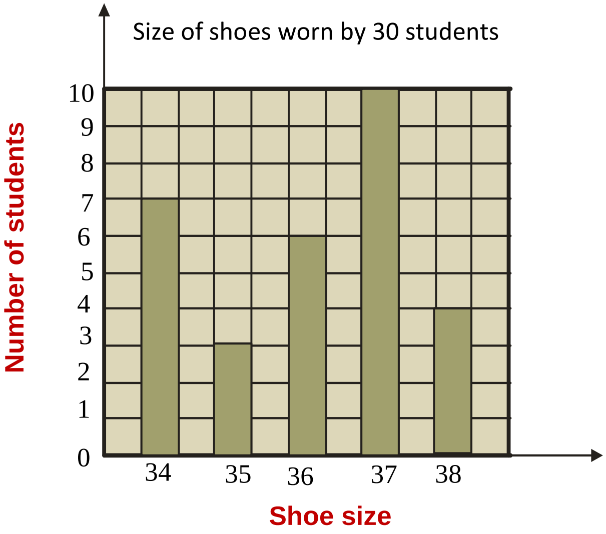


Figure 4.7

You can use the table and the bar graph to answer these questions.

a) What is the most common size of shoe worn by children at the school?

b) Which size is not common in the class?

c) How many different sizes of shoe are worn by the 30 children at the school?

Check your answer with the given solution

Solution: (a) 37 (b) 35 (c) 5

Note: Whenever you draw a bar graph you must have:

- **a title**
- **labels on the horizontal and vertical axes to show what they represent.**

All of these features must be included when drawing a graph, because they are essential when interpreting the graph.

Group work 4.1

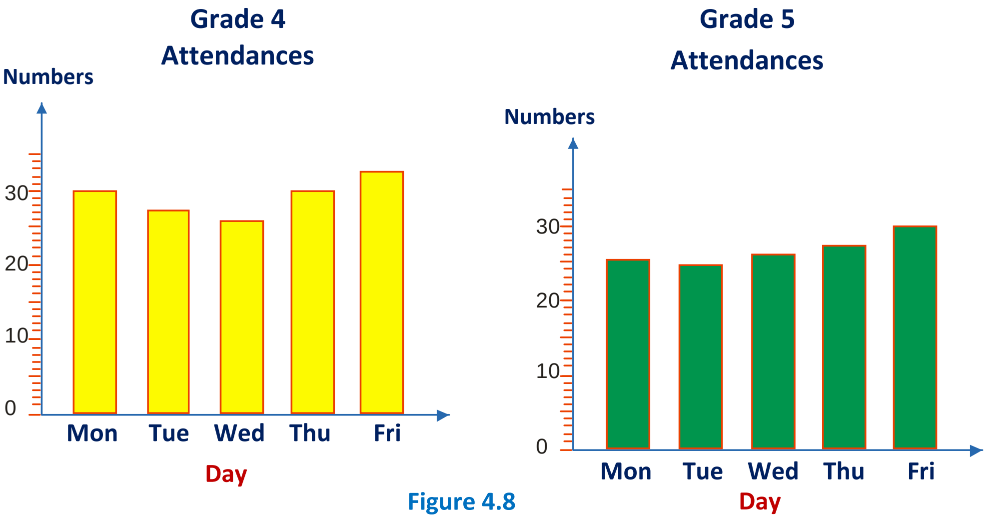


Figure 4.8

The bar graphs show the attendances in grade 4 and grade 5 classes last week.

a) **How many Grade 4 children were in school on:**

1. Monday? ☐ 2. Thursday? ☐

3. Wednesday? ☐ 4. Friday? ☐

b) **How many Grade 5 children were in school on:**

5. Friday? ☐ 6. Wednesday? ☐

7. Tuesday? ☐ 8. Thursday? ☐

c) **Which class had most children on:**

9. Tuesday? ☐ 10. Friday? ☐

11. Wednesday? ☐ 12. Monday? ☐

d) **There are 32 children in Grade 4. How many were absent on:**

13. Tuesday? ☐ 14. Friday? ☐

e) **There are 30 children in Grade 5. How many were absent on:**

15. Monday? ☐ 16. Wednesday? ☐

Example 2

The bar graph shown below represents children's age in a certain village. You can use the bar graph to answer each of the following questions (Remember to check your answer with the solution given).

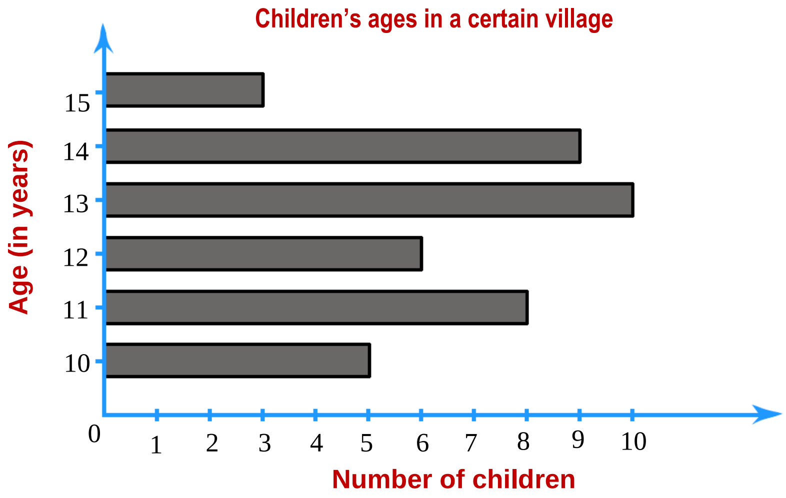


Figure 4.9

a) How many children are 13 years of age?
b) How many children are less than 16 years of age?
c) How many children are over 13 years old?
d) How many children are 10 years old?
e) How many children are under 14 years old?
f) How many children are there altogether in the village?

Solution

a) 10 b) 41 c) 12 d) 5 e) 29 f) 41

Exercise 4A

1. The green club members planted trees around a foot ball field.
   a) How many of each type of tree did they plant?
   b) What was the total number of trees planted?
   c) Which type of tree is most planted?

d) Which type of tree is least planted?

e) The club members want to plant more trees so that there will be the same number of each. How many of each tree should they plant?

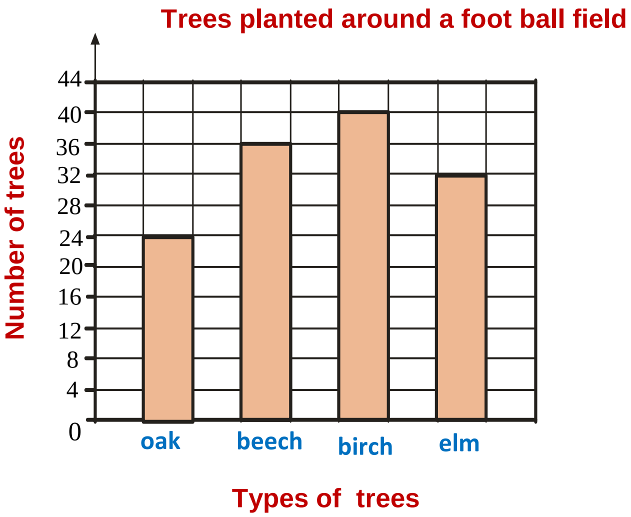
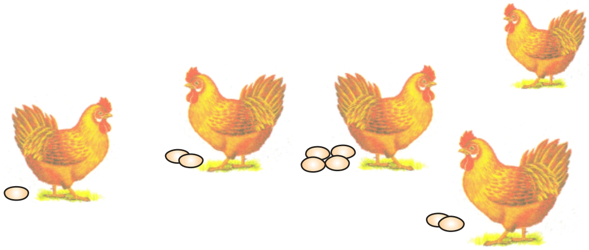


Figure 4.10

2. Abdu had a poultry farm. He collected eggs daily from Monday to Sunday. (Figure 4.12)

In one week he recorded the numbers he collected in this table.

(i) Complete the table by using the bar graph that Abdu draws.



Eggs collected daily in a week

Figure 4.11

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of eggs	**45**					

ii) a) How many eggs were collected on Monday?

b) On which days were 25 eggs collected?

c) On which days were most eggs collected?

d) On which days were least eggs collected?

e) How many more eggs were collected on Saturday than Friday?

f) What was the total number of eggs collected in one week?

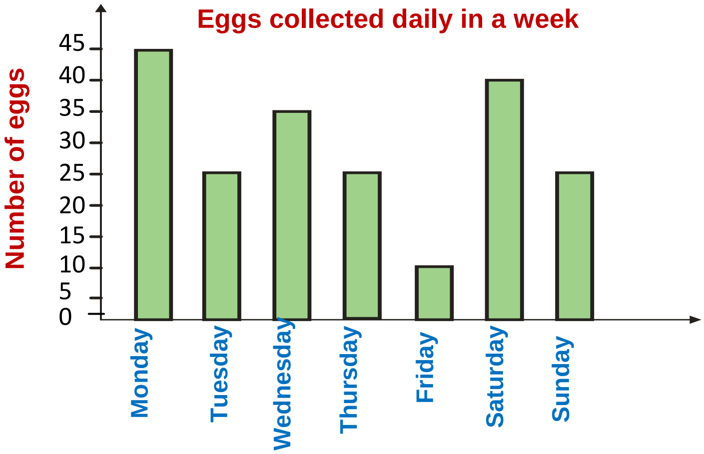


Figure 4.12

3. Eyerusalem's test marks (out of ten) in five subjects were:

English	5
Mathematics	8
Basic science	4
Social studies	7
Music	9

Draw a bar graph showing:

a) the subjects on the horizontal axis

b) marks on the vertical axis

Remember to label the axes and give your bar graph a precise title.

4. Lemlem recorded the marks of her class in a Mathematics test marked out of 10.

4 1 7 6 0 3 8 7 2 4 5 0 3 7
8 9 7 5 0 3 2 1 8 9 3 7 10 1
6 8 9 10 3 7 6 2 5 8 10 7

a) Draw a chart (table)

b) Draw a bar graph

5. Students in grade 5 of a certain school investigate where insects are found. (Figure 4.13)

a) How many insects were found

   (i) On leaves
   (ii) Under stones
   (iii) On flowers?

b) How many more were found in the grass than

   (i) Under stones
   (ii) In the air
   (iii) On flowers?

c) What was the total number of insects found?

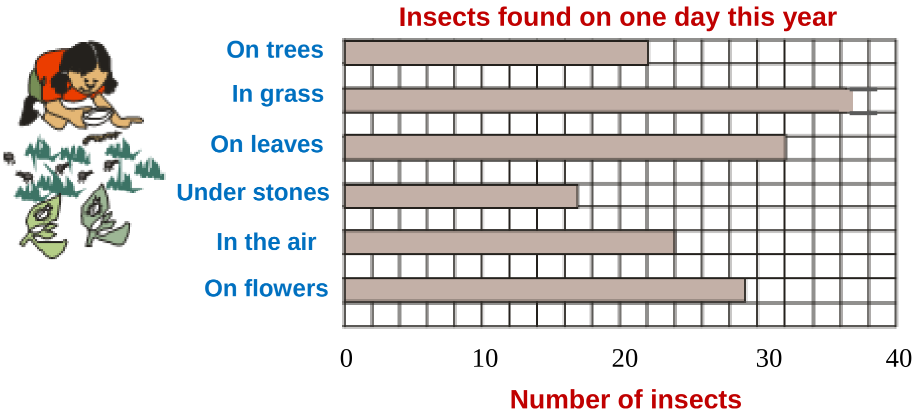
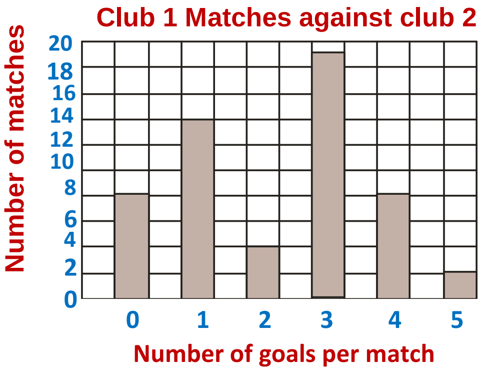


Figure 4.13

6. The bar graph shows goals scored when foot ball club 1 played against foot ball club 2.



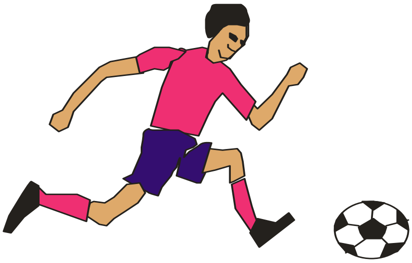


Figure 4.14

a) In how many matches did club 1 score 2 goals?

b) In how many matches did club 1 score more than 3 goals

c) What was the most commonly occurring number of goals scored in the matches?

d) In how many matches did the team score no goals?

e) How many matches were played between club 1 and club 2 altogether?

7. The students in Nega's class have taken a test in mathematics. Here are the results out of 10.

Student's Name	Score	Student's Name	Score
Yishak	7	Abeba	8
Omer	6	Danayt	6
Aman	4	Aklil	5
Obang	5	Ashkuti	6
Naod	8	Konjit	6
Semira	6	Yaregal	5
Senayt	6	Sofia	2
Hana	4	Ekubay	7
Kelemua	6	Kasim	7
Yalew	9	Shentema	6
Derartu	7	Habib	8
Mahlet	6	Amare	10
Yonas	9	Dilbo	7
Nega	10	Yafet	5
Aregawi	7	Siyane	9

Complete the following table by putting the data given in Nega's class.

Marks scored in mathematics test											
Score	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number of students	**0**	**0**	**1**	**0**	**2**						

Draw a bar graph showing 'score' on the horizontal axis and 'number of students' on the vertical axis.

4.2. The Average of Numbers

Activity 4.4

The table represents marks of six subjects of a student in grade five in first semester.

Subject	Marks out of 100
Civics	70
English	80
Maths	60
Basic science	90
Social study	50
Sport	70

- **Add the marks**
- **Divide the total mark by 6.**
- **Write the result**
- **What do you call this result?**
- **What is the student's average mark?**

In this sub-unit you will deal with finding the average of numbers.

The **average** is found by adding the values of the data and dividing by the total number of values. or average $= \frac{\text{Total number of value}}{\text{number of values}}$. For example, the **average** of 3, 2, 6, 5 and 4 is found by adding $3 + 2 + 6 + 5 + 4 = 20$ and dividing by 5; hence the average of the data is $20 \div 5 = 4$.

Definition 4.1: The average of numbers is the sum of the values, divided by the total number of values.

Example 3

The chart shows how many students took part in sport activities. You can read from the given data that most students took part in foot ball and fewest took part in volley ball. Find the average number of students who took part in sport activities.

Activities	Number of students
Foot ball	72
Tennis	40
Basket ball	48
Volley ball	24
Fast walking	60
Running	56

Figure 4.15

Solution:

$$\text{The total number of students} = 72 + 40 + 48 + 24 + 60 + 56$$
$$= 300$$

$$\text{Average number of student in sport activities} = \frac{\text{total number of students}}{\text{total number of sport activities}}$$

$$\text{Average} = \frac{300}{6}$$

$$\therefore\ \text{Average} = 50$$

Example 4

Students in Grades 1 – 6 collected money for seedlings. Find the average amount of money collected.

Solution: You can use table to describe the bar graph easily as follows.

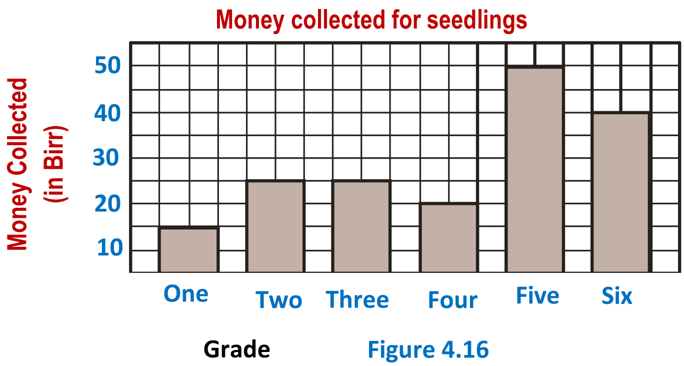


Figure 4.16

Money collected for seedlings						
Grade	**One**	**Two**	**Three**	**Four**	**Five**	**Six**
Amount (in Birr)	**15**	**25**	**30**	**20**	**50**	**40**

$$\text{Average amount of money collected} = \frac{\text{total amount of money collected}}{\text{total number of grades}}$$

$$\text{Average} = \frac{15+25+30+20+50+40}{6} = \frac{180}{6}$$

$\therefore$ average = Birr 30

When you work out problems that involve travel, you need to find the speed:

$$\text{Speed} = \frac{\text{distance}}{\text{time taken}}$$

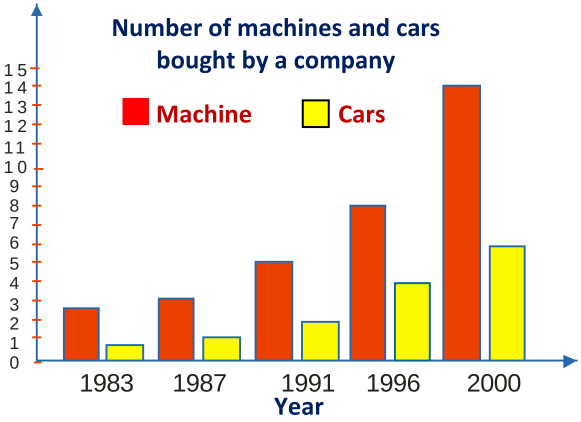
Since the speed probably varies over the whole journey, we usually consider the **average** speed:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time taken}}$$

Group work 4.2

The bar graph given below shows the number of machines and cars bought by a company.

1. Find the average number of machines bought by the company in five years
2. Find the average number of cars bought by the company in five years.



Example 5

The driver of a lorry covered a distance of 200 kilometers in 4 hours. What was his average speed?

Solution

Distance covered = 200 kilometers

Time taken = 4 hours

Average speed $= \frac{\text{total distance}}{\text{total time taken}} = \frac{200}{4} =$ 50 km per hour

Exercise 4B

1. The ages of 20 students in a class were recorded as follows. 12, 13, 12, 13, 12, 12, 10, 15, 14, 10, 16, 12, 13, 14, 10, 11, 12, 13, 14, 15 organize this information in a table showing ages and number of students. Find the average age of the students.
2.

Name of students	Test score of 6 students out of 10			
	Test 1	Test 2	Test 3	Test 4
Alexander	8	7	6	9
Kelifa	9	5	7	8
Mihiret	6	8	7	5
Dejenie	4	5	6	7
Bosena	6	4	5	6
Merima	10	8	9	9

Use the above table of datas to answer each of the following questions.

a) What is Alexander's average test score?
b) What is Bosena's average test sore?
c) What is Merima's average test score?
d) What is the average test score of students in Test 1?
e) What is the average test score of students in Test 3?

3. What should be the value of x if the average of the numbers 2, 4, 6, 5 and x is 10?

4. This table shows the rainfall at a certain town from January to August

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Rain fall(mm)	10	15	5	10	20	20	15	10

What was the average rainfall?

5. Shewaye rode her bicycle for three hours from Town A to Town B which is 9 kilometers long. What was Shewaye's average speed?

6. The table below shows visitors of a certain place in a week.

Days	Mon	Tues	Wed	Thur	Fri	Sat	Sun
Number of visitors	64	73	70	80	84	90	120

a) Find the average number of visitors per day.

b) On which days was the number of visitors above average?

7. The bar graph shows the points scored by each player in a high school basket ball game.

a) How many players scored over 10 points?

b) Find the average of the points scored.

c) How many players scored below average?

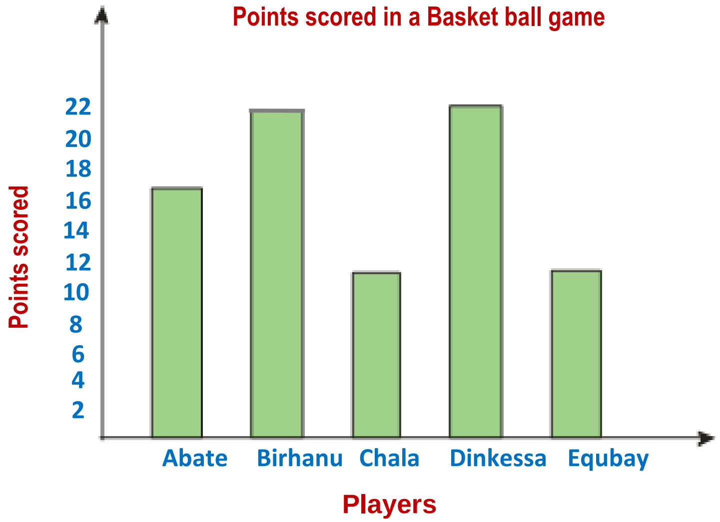


Figure 4.17

UNIT SUMMARY

Important facts you should know:

- **Data handling** deals with collecting, organizing, and summarizing numerical facts. When the data is collected and displayed in a graph, you can look for trends and study details of the data.
- A **bar graph** is a pictorial representation of numerical data by a number of bars of uniform width erected vertically or horizontally with equal spacing between the bars.
- Whenever you draw a bar graph you must have:
  - a title
  - labels on the horizontal and vertical axes to show what they represent
- The **average** of numbers is the sum of the values, divided by the total number of values.

REVIEW EXERCISE

1. Identify whether each of the following statements is true or false.
   a. The fewest number of students absent were on Tuesday and Thursday.
   b. The highest number of students absent was on Monday.
   c. The total number of students absent on the week days is equal to 6.

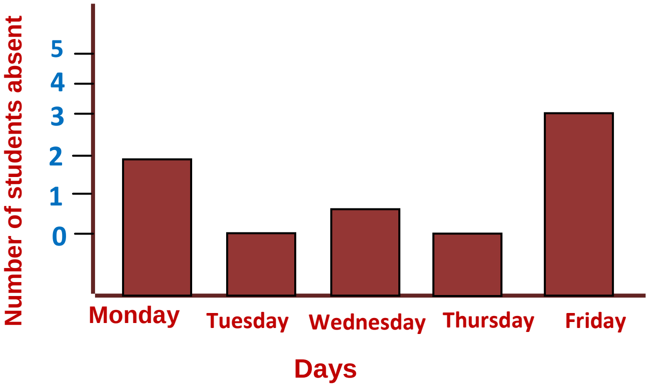


Figure 4.18

2. Children in a certain village investigate where insects are found.

   a. Draw a bar graph using the information in this table.
   b. What was the total number of insects found?
   c. How many more were found in the grass than
      (i) on leaves (ii) on trees

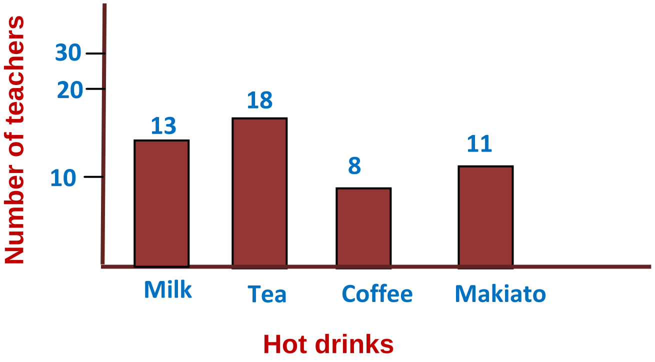
Insects found

On trees	28
In grass	36
On leaves	32
under stones	18
In the air	23
On flowers	16

Figure 4.19

3. The scores of a group of college students is given as: 98, 100, 84, 88, 92, 96, 90, 78, 50, 61, 89, 85, 75. Find the average score. How many of the scores are greater than average score?

4. The following bar graph shows the number of teachers who ordered hot drinks during tea break in a given morning. What is the percentage of teachers who ordered Coffee?



A. 16%
B. 18%
C. 8%
D. 22%

Figure 4.20

UNIT 5

GEOMETRIC FIGURES AND MEASUREMENT

Unit outcomes: After completing this unit you should be able to:

- know important properties of axial symmetry and use this knowledge for carrying out constructions.
- bisect line segments and angles.
- know the unit "degree" and measure the size of a given angle.
- understand and apply the formulas used to compute the areas of rectangles and squares.

Introduction

In this unit you will be introduced to the basic concepts of geometry and measurement. You will study about construction, bisecting line segments and angles, measuring angles and also computing the areas of rectangles and squares.

5.1. Lines

Here you will study abut construction of intersecting and parallel lines, bisecting a given line segment, and construction of perpendicular line to a given line.

An important topic in geometry is construction. You will need a ruler, a pair of compasses and a sharp pencil. It is very important that you use a hard pencil, with a sharp point, otherwise you will not be able to be sure that lines cross accurately, and this can affect the lengths you measure.

5.1.1. Construction of intersecting and Parallel Lines

Activity 5.1

Think of a point of intersection that you passed on your way to school today.

How many roads or paths meet there?

Draw the roads or paths.

Mark the point of intersection.

For each diagram: how many lines are there? How many intersections are there?

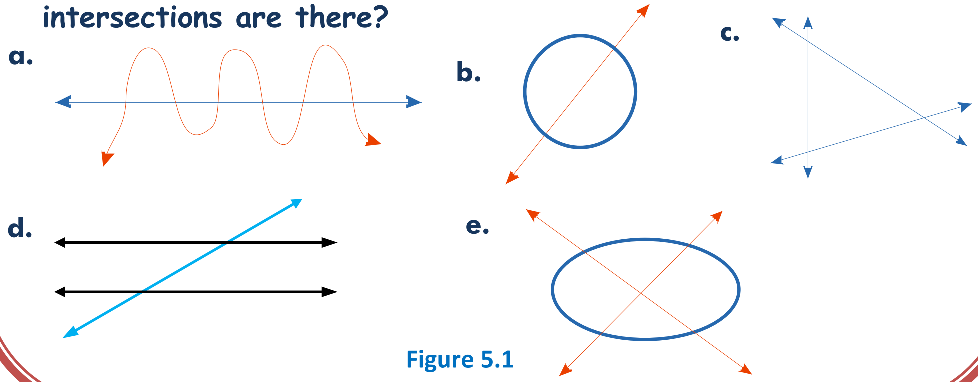
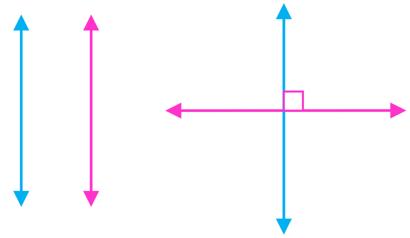


Figure 5.1

Remember that a plane is an infinite flat surface. A line is a series of points that extends in two opposite directions without end. Lines in a plane that never meet are called **parallel** lines. Lines that intersect to form a right angle (90°) are called **perpendicular** lines. Intersecting lines have exactly one common point.



Parallel lines

Perpendicular lines

Figure 5.2

A line segment is formed by two endpoints and all the points between them.

Line segment

Figure 5.3

Example 1

Use the figure to name a line segment, a point, two intersecting lines, and a pair of parallel lines.

Solution. **Two endpoints are S and U, so they form a line segment, $\overline{SU}$.**

There are 5 points, R, S, M, U, T. Intersecting lines have exactly one point in common.

So, $\overleftrightarrow{RU}$ and $\overleftrightarrow{SU}$ are intersecting lines.

$\overleftrightarrow{TU}$ never intersects $\overleftrightarrow{RS}$, so $\overleftrightarrow{TU}$ and $\overleftrightarrow{RS}$ are parallel lines.

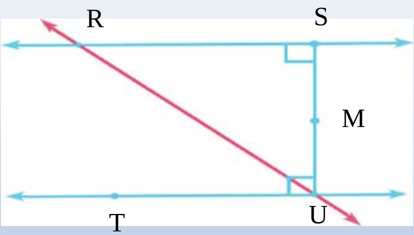


Figure 5.4

Group work 5.1

You can use a ruler and set square to draw parallel line to a given line AB through another point D that is not on the given line as follows:

Step 1. Slide the set square along AB until the short side passes through D.

Step 2. Draw a line along the short side of the set square. Then slide the set square up the line you have just drawn using your ruler, until it reaches D. Now draw a line along the long side of the set square.

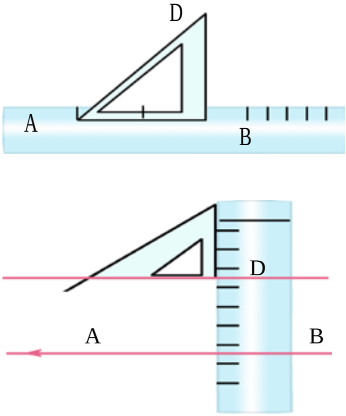


Figure 5.5

Exercise 5.A

1. Use the figure to name each of the following.
   a) A line segment ___________
   b) A point ______________
   c) Two pairs of intersecting lines ________
   d) A pair of parallel lines ____________

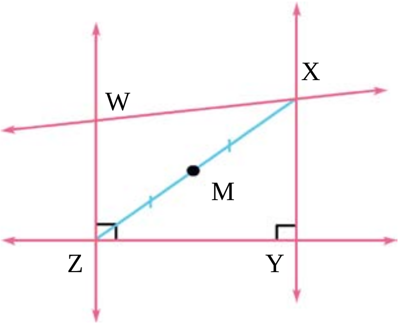


Figure 5.6

2. Drawing parallel line
   a. Draw a straight line
   b. Put the side of a set square along your line, and lay a ruler along the base, like this.

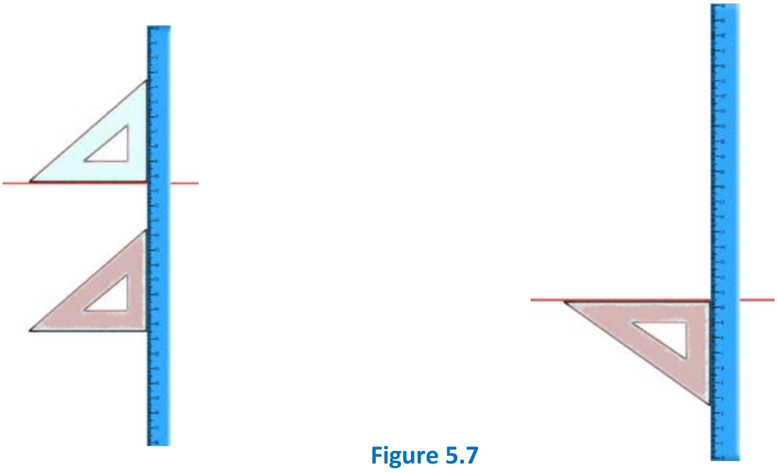


Figure 5.7

   c. Hold the ruler very still, and slide the set-square along the ruler for about 10cm. Hold the set-square very still and draw along it, to make a line parallel to the first line.
   d. Now draw some more sets of parallel lines, as follows.
      a. A pair of lines 6 cm apart.
      b. A pair of lines 12 cm apart.

5.1.2. Bisecting a given Line Segment

To **bisect** an angle or a segment means to separate it into two congruent parts. Here you will study how to bisect a segment.

You can use paper folding methods to bisect a given segment.

Group Work 5.2

Work with a partner.

Materials: compass, ruler, paper.

Bisect a line segment using paper folding.

- Use a ruler to draw $\overline{MN}$.
- Fold point N onto point M and make a crease as shown. The crease bisects $\overline{MN}$. Label the intersection point L.

Figure 5.8

Discuss

Measure $\overline{ML}$ and $\overline{LN}$. What can you say about point L?

Using a pair of compasses

1. Make sure the pencil is sharp.
2. Fit the pencil into the compasses.
3. Close the compasses and make sure that the needle point and the pencil tip are close together.
4. Tighten up the clip holding the pencil in place.
5. Use a ruler to set the radius of the compasses. Put the needle point on the zero of the ruler and pull the compasses apart until the pencil points is at the correct measurement for the radius required.
6. Turn the compasses around, with the pencil point at the zero, and check that the needle point is at the correct measurement.

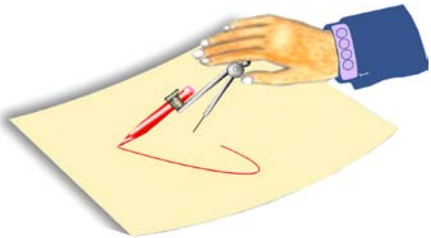


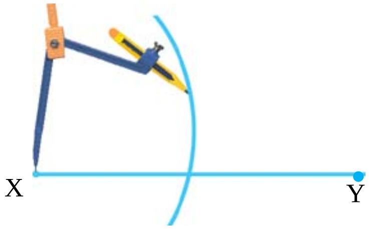
Figure 5.9

Activity 5.2

Draw a segment and bisect it.

- Use a ruler to draw a segment. Label the endpoints X and Y.

- Open your compass to a setting that is longer than half the length of $\overline{XY}$. Place the compass point at X and draw a large arc.



- Using the same setting, place the compass point at y and draw a large arc to intersect the first arc twice.

Figure 5.10

- Use a ruler to draw a segment connecting the two intersection points. This segment intersects $\overline{XY}$. Label this point Z.

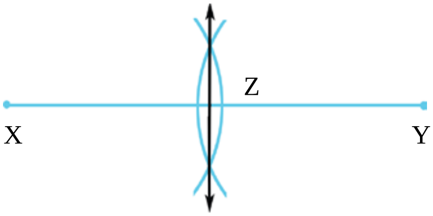


Figure 5.11

What do you think?

1. Use the compass to measure the distance from X to Z. compare this to distance from Z to Y. What do you find?
2. How is $\overline{XZ}$ related **to** $\overline{ZY}$**?**
3. How is the segment you drew through Z related to $\overline{XY}$?

Exercise 5.B

Draw line segment with the given measurement. Then use a ruler and compass to bisect each segment.

a) 8cm b) 10 cm c) 13 cm d) 16 cm

5.1.3. Construction of Perpendicular Line to a Given Line

Remember that **perpendicular lines** are lines in the same plane that form right angles when they intersect. In the figure, line ℓ is perpendicular to line m. this can also be written as ℓ ⊥ m. Two ways to construct perpendicular lines are described below.

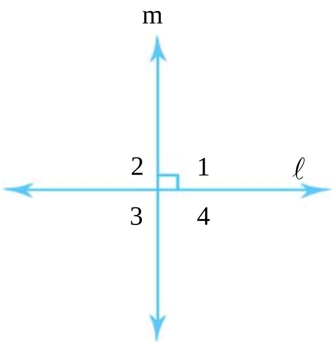
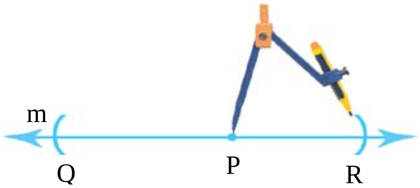


Figure 5.12

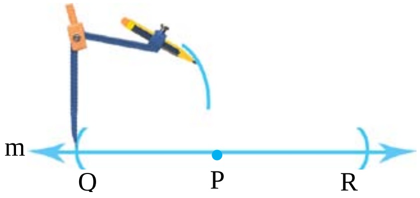
Activity 5.3

Construct a line perpendicular to line m through point p on m.

- Draw a line and label it m. Draw a dot on the line and label it point p.
- Place the compass point on p and draw arcs to intersect line m twice. Label these points Q and R.



- Open your compass wider. Put the compass at Q and draw an arc above line m.



- With the same setting, put the compass at R and draw an arc to intersect the one you just drew. Label this intersection point S.

- Use a ruler to draw a line through S and P.

  By construction, $\overleftrightarrow{PS} \perp m$.

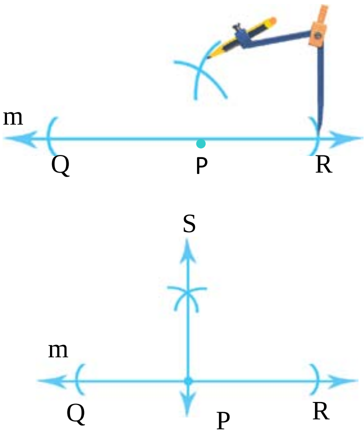


Figure 5.13

Activity 5.4

1. Construct a line perpendicular to line m through point p not on m.
   - Draw a line and label it m. Draw a dot above m and label it point P.
   - Open the compass to a width greater than the distance from p to m. Draw a large arc to intersect m twice. Label these points of intersection Q and R.

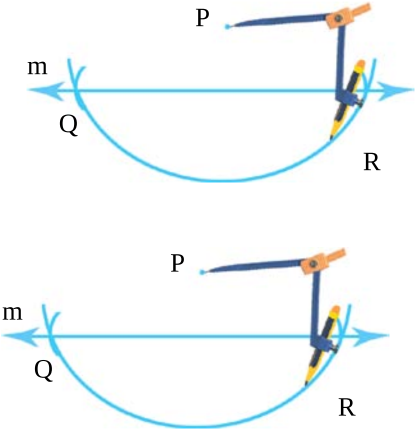
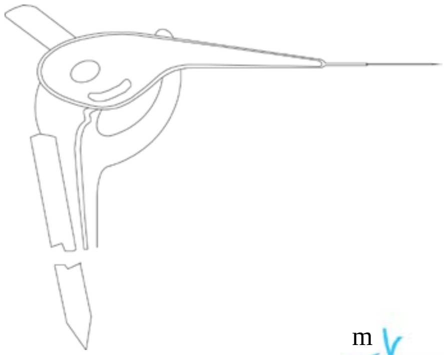


Figure 5.14

   - Put the compass at Q and draw an arc below m.



- Using the same setting, put the compass at R and draw an arc to intersect the one drawn from Q. Label the intersection point S.

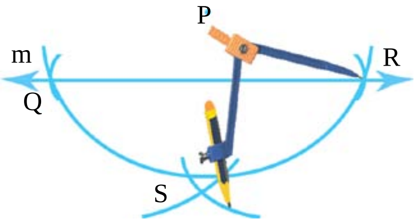


Figure 5.15

- Use a ruler to draw a line through P and S.

  By construction, $\overleftrightarrow{PS} \perp m$.

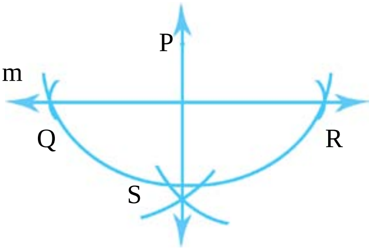


Figure 5.16

What do you think?

What type of angles are formed by perpendicular lines?

2. You can construct a square by using perpendiculars.
   a. Draw line ℓ. Draw two dots on ℓ and label them points R and S. Construct a perpendicular through R.
   b. Use the compass to measure the distance from R to S. using the same setting, place the compass at R and draw an arc on the perpendicular through R. Label this point T.

c. Using the same setting, place the compass at T and draw an arc to the right of T, then place the compass at S and draw an arc to intersect the one you just drew. Call this point U.

d. Use a ruler to draw $\overleftrightarrow{TU}$ and $\overleftrightarrow{US}$ Figure RSUT is a square.

Exercise 5.C

1. Trace these diagrams. In each case, drop a perpendicular to the line from the point.

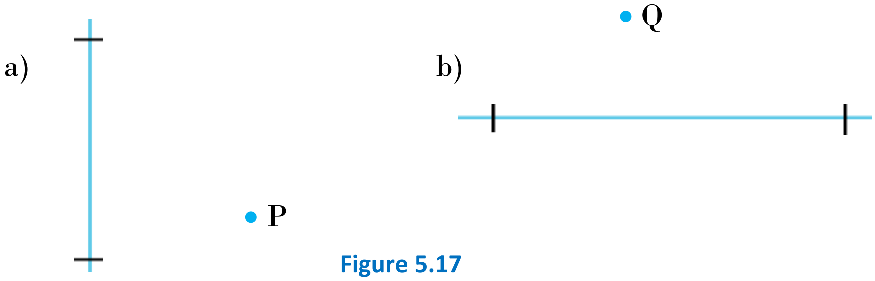


Figure 5.17

2. Draw a square of side 4 cm.
3. Draw a rectangle of sides 3 cm and 2 cm.

5.2. Angles and Measurement of Angles

In this sub- unit you will learn about angles, classification of angles, measurement of angles and bisecting angles.

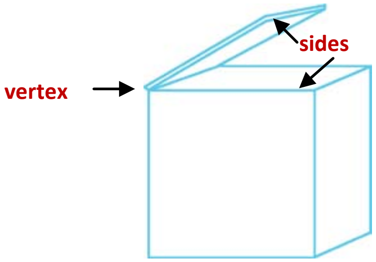
5.2.1. Angles

When Abrham built his new home, an oak tree on the lot was cut down. Ato Abrham wishes to cut and split the oak logs to burn in their fire place. He will use a wedge to split the logs.

Figure 5.18

From the front, the **sides** of a wedge look like two lines that meet in a point called the **vertex**. Other examples of wedges are needles and ski jumps.

The vertex and sides of the wedge form an angle. The sides of an angle may be opened like a box lid. An angle is large or small according to the amount of openness.



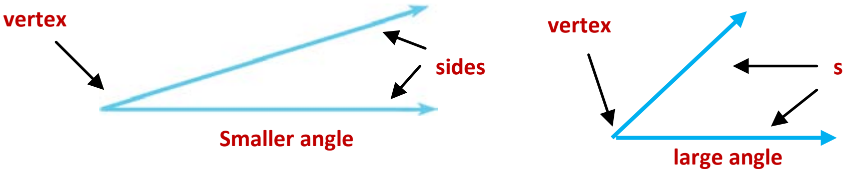


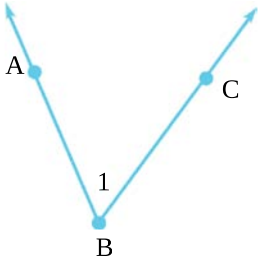
Figure 5.19

Definition 5.1: When two segments or rays have a common end point, they form an **angle**. The point where they meet is called the **vertex** of the angle.

Note

1. An angle can be named by its vertex. To say an angle with vertex B, we write ∠ B. Angles can also be named using a point from each side and the vertex, ∠ ABC or ∠CBA. The vertex letter always goes in the middle. Another way to name an angle is to use a number inside the angle, ∠1.

2. The arrows on the sides of the angle tell you that you can extend the sides.



∠ B or ∠ ABC
or ∠ CBA or ∠ 1

Figure 5.20

Exercise 5.D

1. Name the angle, the vertex and sides of the angles shown below.

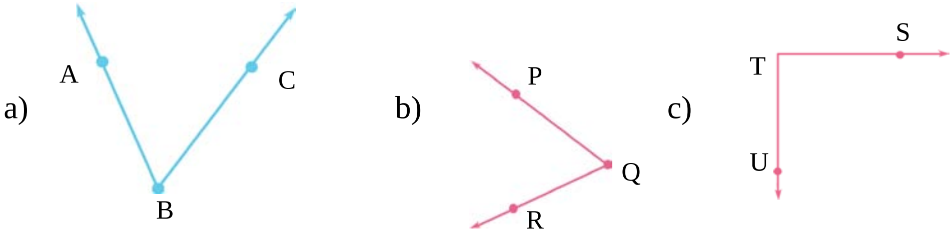


Figure 5.21

2. What is the measure of <DBA? <ABC? <DBC?

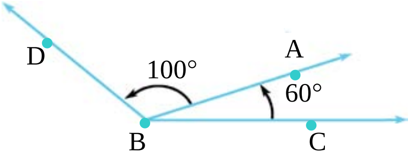


Figure 5.22

5.2.2 Measurement and Classification

The most common unit used in measuring angles is **degree**. Imagine a circle cut in to 360 equal-sized parts. Each part would make up a one-degree (1°) angle as shown.

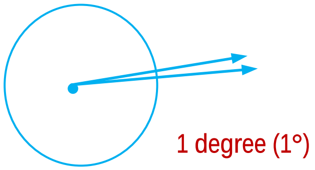


Figure 5.23

Note: An angle is not measured by the length of its sides. You can use a **protractor** to measure **angles.**

1. Place the centre of the protractor on the vertex (B) of the angle with ruler along one side.
2. Use the scale that begins with 0° on the right side of the angle. Read the angle measure where the other side crosses the same scale. Extend the sides if needed.

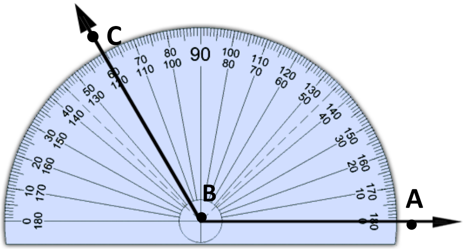
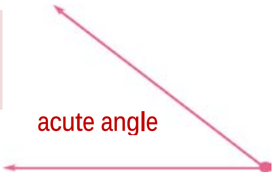


Figure 5.24

The angle measures 120°. That is, $m(\angle ABC) = 120°$
or $m\left(A\hat{B}C\right) = 120\,°$ or $m\left(\hat{B}\right) = 120°$

Note: $m(\angle B)$ or $m(\hat{B})$ means the measure of angle B.
Angles can be classified according to their measure.

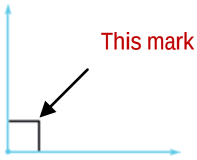
Definition 5.2: An Acute angle is an angle whose measure is between 0° and 90°.



between 0° and 90°

Figure 5.25

Definition 5.3: A right angle is an angle whose measure is 90°.

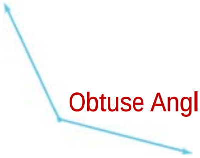


exactly 90°

Figure 5.26

For example, the corner of a picture frame is a right angle. Its measure is 90°.

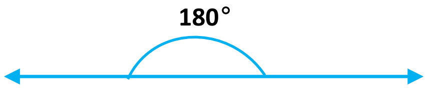
Definition 5.4: An obtuse angle is an angle whose measure is greater than 90° but less than 180°.



between 90° and 180°

Figure 5.27

Definition 5.5: A straight angle is an angle whose measure is 180°.



a straight angle

Figure 5.28

Note: An angle on a straight line is a straight angle.

Definition 5.6: A reflex angle is an angle whose measure is greater than 180° but less that 360°.

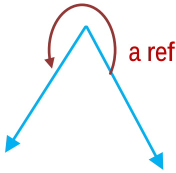


Figure 5.29

Group Work 5.3

Work with a partner

Materials: protractor, and ruler.

- Draw any angle. Place the protractor on the angle so that the centre is on the vertex of the angle and the 0° line lies on one side of the angle.
- There are two scales on your protractor. Use the one that begins with 0° where the side aligns with the protractor.
- Follow the scale from the 0° point to the point where the other side of the angle meets the scale. This is the angle's measure.

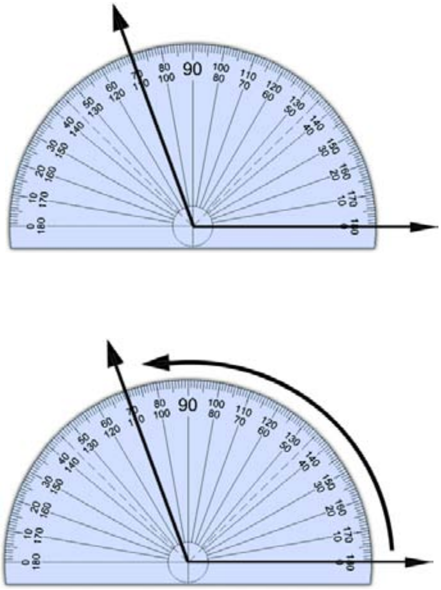


Figure 5.30

Discuss

1. How do you know which scale to read?
2. What do you need to do if the sides of your angle do not intersect the scale of the protractor?
3. Kebede says the angle above has a measure of 70°. What's wrong?
    - You can use a protractor to draw an angle of a given measure. Suppose you want to draw a 65° angle.
        a. Draw a line segment.
        b. Align your protractor on the segment with the centre on one end point of the segment.
        c. Find the scale that starts with 0°. Go along that scale until you find 65°. Put a mark at this point.
        d. Draw a line through the end point of the segment and the mark.

Group work 5.4

Work with a partner.

Materials: round paper plate, scissors, protractor.

Figure 5.31

- Find the centre of the plate by folding it in half twice.
- Cut a right-angle wedge along the fold lines.
- Cut acute wedge from the plate.
- Cut an obtuse wedge from the plate.
- Use a protractor to measure the angle formed by each wedge.

Discuss

How could you use the right angle wedge to determine if an angle is acute or obtuse?

Example 2

Let us use a protractor to find the measure of each angle and classify each angle as acute, right or obtuse.

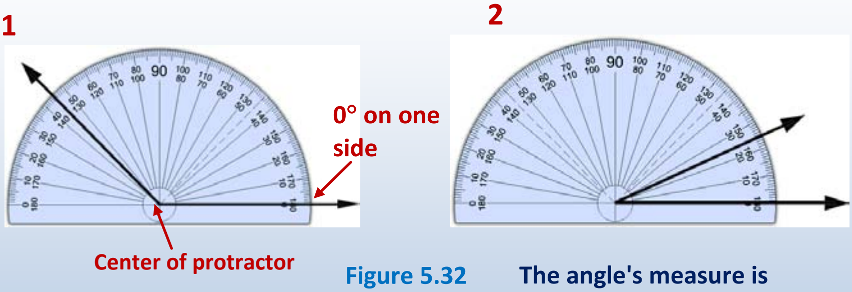


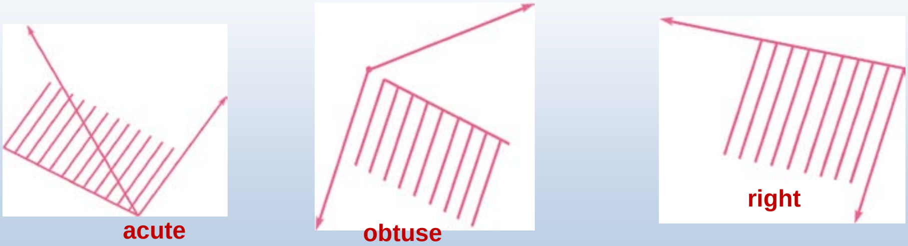
Figure 5.32

The angle's measure is 135°. It is obtuse angle.

The angle's measure is 25°. It is an acute angle.

Example 3

Lay one corner of your notebook paper on top of each angle to determine whether each angle is acute, obtuse or right.



acute obtuse right

Figure 5.33

Example 4

Draw a 150° angle.

Solution

Step i)

Draw one side the vertex and draw an arrow.

Step ii)

Find 150° on the appropriate scale. make a pencil mark

Step iii)

Draw the side that connects the vertex and the pencil mark.

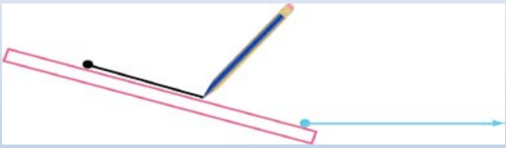


Figure 5.34

Exercise 5.E

1. Use a protractor to find the measure of each angle.

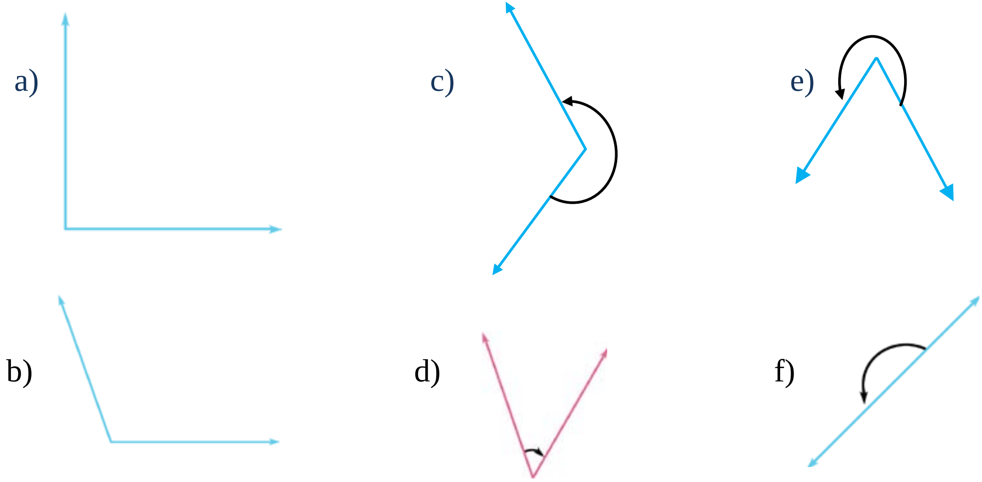


Figure 5.35

2. Classify each angle as acute, right, obtuse or reflex

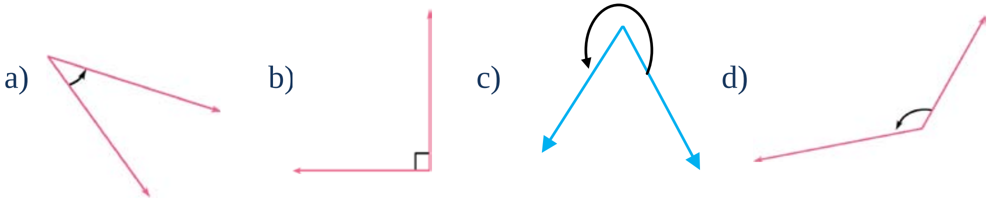


Figure 5.36

3. Classify angles having each measure as acute, right, obtuse straight, or reflex.

a) 99° c) 90° e) 270° g) 114°

b) 27° d) 180° f) 2°

4. An angle measures 90.5°. Is it an obtuse angle or a right angle?

5. Use a protractor to draw angles having the following measure.

a) 75° b) 130° c) 210° d) 90° e) 170°

6. Through what angle does the minute hand of a clock turn in 5 minutes?

7. The branches on young trees should be spread to form angles of at least 60° with the tree trunk. This strengthens branches and allows for more air circulation and light. Which branches on the tree at the right need to be spread?

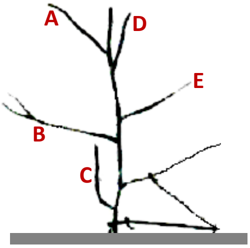


Figure 5.37

8. What is the measure of <MZN?
<NZO? <PZO?

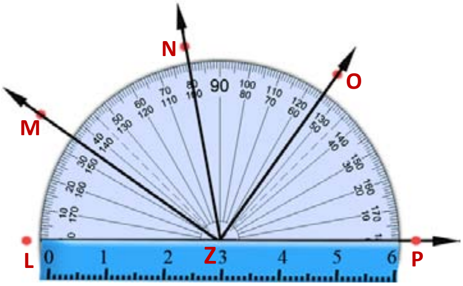


Figure 5.38

5.2.3 Bisecting an Angle

Definition 5.7: If <A has the same measure as <B, then <A is congruent to <B.
In symbols: If m(<A)=m(<B), then <A ≅ <B.

Note: ≅ means is congruent to.

When you separate an angle in to two congruent angles, you bisect the angle. In Figure 5.39

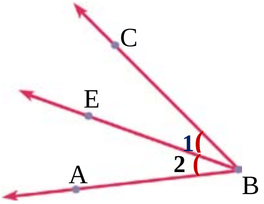


Figure 5.39

$\overrightarrow{BE}$ bisects <ABC.

So m(<1) =m(<2).

This means that <1 ≅ <2.

Group work 5.5

Work with a partner

Materials: Ruler, protractor

- Use your ruler to draw any angle.
- Fold the paper through the vertex so that the two sides match when you hold the paper up to the light.
- Unfold the paper and use your ruler to draw a segment on the fold.

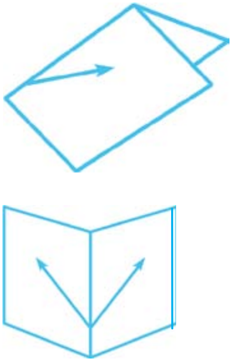


Figure 5.40

Discuss

a. Use your protractor to measure the original angle. Then measure the two smaller angles.

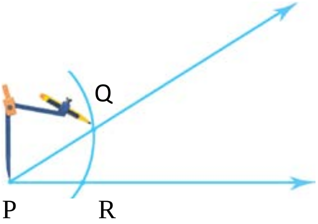
b. Write a sentence to relate the measures of the smaller angles to that of the larger one.

Now, let us study how to bisect an angle.

Activity 5.5

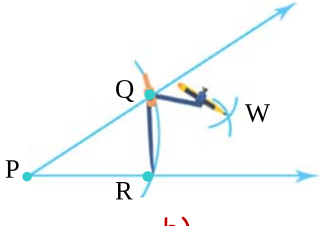
Draw an angle and bisect it.

1. Use a ruler to draw any angle. Label the vertex p. Place the compass at the vertex and draw a large arc to intersect each side. Label the intersection points Q and R.



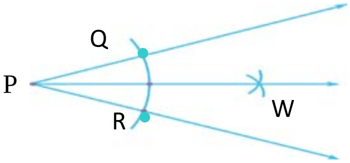
a)

2. Place the compass at Q and draw an arc on the inside of the angle. Using the same setting, place the compass at R and draw an arc to intersect the one you just drew. Label the intersection point W.



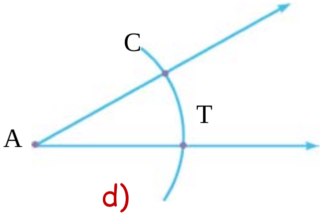
b)

3. Draw ray PW.
4. Use your protractor to measure ∠QPW and ∠WPR. What do you find?



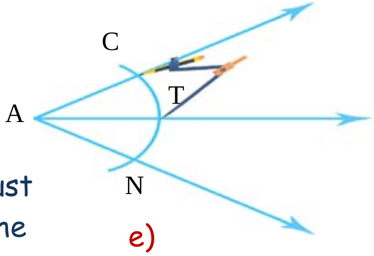
c)

5. How is ∠QPW related to ∠QPR?
6. Suppose you are given an angle and told that its measure is half that of a larger angle. How would you construct the larger angle?
    a. Draw any angle and label it ∠A.



d)

    b. Draw an arc through the sides of the angle into the outside of the angle. Label the intersection points C and T.



e)

Figure 5.41

    c. Put the compass point at T. Adjust the setting so that it measure the distance from T to C. Without removing your compass, draw an arc to intersect the large arc outside of ∠A. Call this point N.
    d. Draw $\overrightarrow{AN}$.$\overrightarrow{AT}$ is the bisector of ∠CAN.
    e. Complete: m (∠CAT)= _______ m(∠CAN)

Example 5

In the figure shown, $\overrightarrow{BA}$ bisects <DBC. Use an equation to find the value of x if the measure of <DBC is 140°.

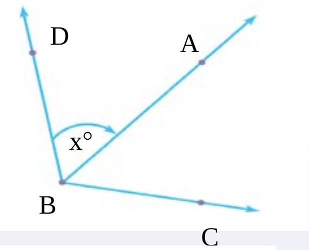


Figure 5.42

Solution: $\overrightarrow{BA}$ **bisects ∠DBC. So, m(∠DBA) = m(∠ABC).**

Since m(∠DBA)=x, it follows that m(∠ABC) =x.

So, $x + x = 140°$

$$2x=140°$$

$$x = 70°$$

∴ The value of x is 70.

Exercise 5.F

1. Draw the angle with the given measurement. Then use a ruler and compass to bisect each angle.
   a) 50° b) 130° c) 87° d) 90°

2. Use ruler and compass to bisect each of the following angles shown below.

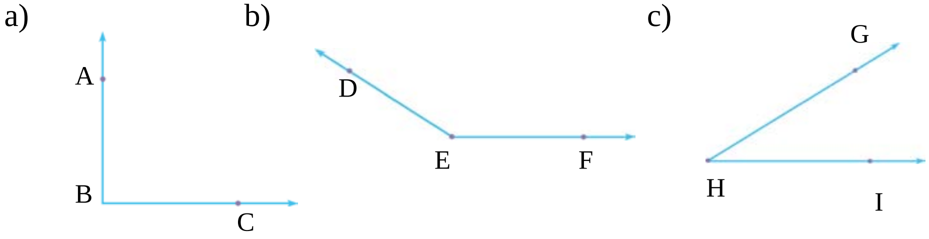


Figure 5.43

3. In the figure shown, $\overrightarrow{XZ}$ bisects ∠WXY. Find the value of a or m(∠YXZ) if m(∠WXY)=124°.

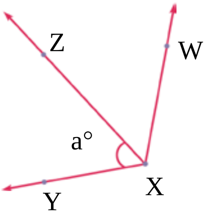


Figure 5.44

5.3. Classification of Triangles

Group Work 5.6

Work with a partner.

- One student should make a triangle on the geo board. A sample is shown at shown at the right.
- Have the partner draw the triangle on dot paper and cut out the triangle.
- Continue this activity until you have ten different triangles. Try to make a variety of triangles.

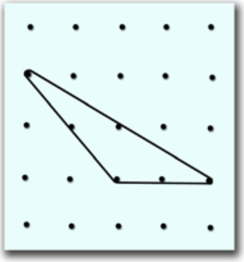


Figure 5.45

Discuss

One way to name triangles is by their angles. At least two angles of every triangle are acute. Sort your triangles into three groups, based on the third angle.

A second way to name triangles is by their sides. Can you sort your triangles in to three groups based on the length of sides?

Remember that a triangle is a three sided closed figure made of three line segments. In the figure shown below, ABC is a triangle. It is written as ⊿ABC. ∆ABC has three sides namely AB, BC and AC. It has three vertices A, B and C. The angles included between two sides are angles of the triangle. ∠ABC, ∠BAC and ∠ACB are three angles of ∆ABC.

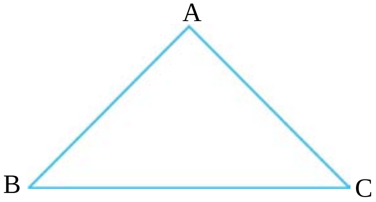
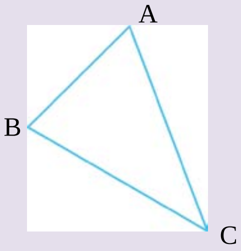
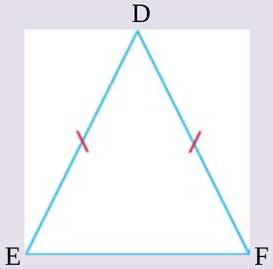


Figure 5.46

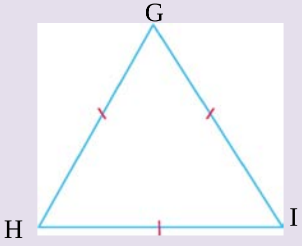
- **A triangle is sometimes classified by the number of congruent sides it has as follows:**



1. **Definition 5.8:** A scalene triangle is a triangle with no sides congruent.
∆ ABC is scalene because AB ≠ AC, AB ≠ BC and AC ≠ BC.



2. **Definition 5.9:** An isosceles triangle is a triangle with at least two sides congruent.
∆DEF is isosceles because DE=DF.



3. **Definition 5.10:** An equilateral triangle is a triangle with all sides congruent.
∆GHI is equilateral because GH=HI=GI.
Is ∆ GHI isosceles? Why?

Figure 5.47

Triangles can also be classified by their angles as follows:

1. **Definition 5.11:** An acute angled triangle is a triangle with three acute angles.

∆ JKL is acute angle because the measures of all the three angles (<J, <K and <L) are between 0° and 90°.

2. **Definition 5.12:** A right angled triangle is a triangle with the measure of one of its angle right (or 90°).

∆ MNO is right angled because m(<N)=90°.

Can there be any other right angle in ∆MNO? why?

3. **Definition 5.13:** An obtuse angled triangle is a triangle with the measure of one of its angles obtuse.

∆PQR is an obtuse angled because m(<Q) is between 90° and 180°.

Can there be any other obtuse angle in ∆PQR? Why?

4. **Definition 5.14:** An equiangular triangle is a triangle with all its three angles congruent.

∆ STU is equiangular because m(<S)= m(<T)= m(<U).

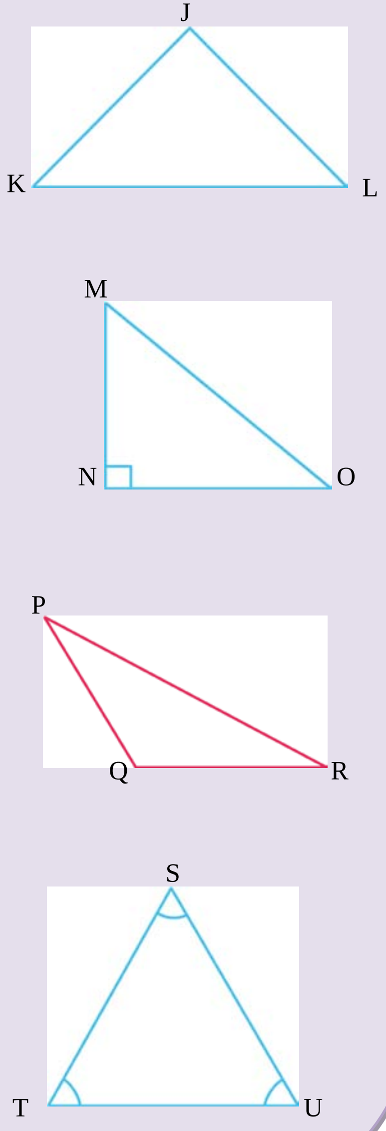


Figure 5.48

Note: The sum of the measures of the three angles of a triangle is 180° (why?)

What is the measure of each angle of an equiangular triangle?

Exercise 5.G

1. Classify each triangle by its sides and by its angles.

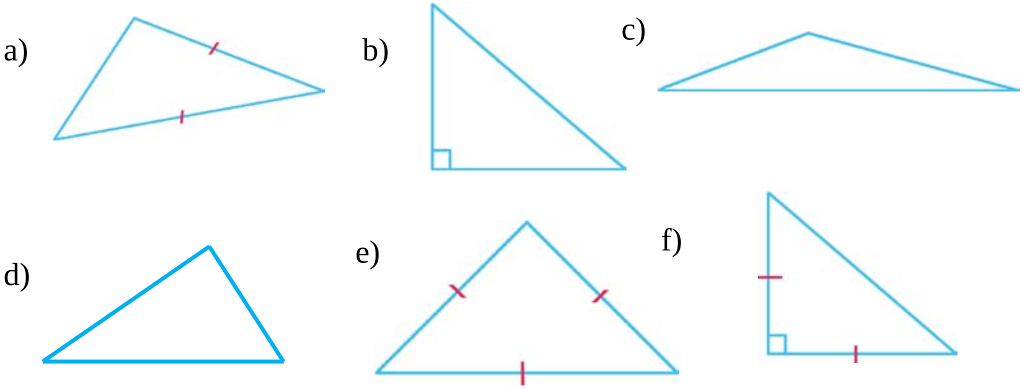


Figure 5.49

2. Make a model. Use square paper.
   - a) To make a right angled triangle.
   - b) To make an isosceles triangle.
   - c) To make an acute angled triangle.
   - d) To make an equilateral triangle.
   - e) To make an obtuse angle triangle.
   - f) To make a right angled isosceles triangle.

5.4. Lines of Symmetry

Did you know that there are more than 15,000 different species of butterflies? The bright colours and attractive patterns make the butterfly one of the most beautiful insects.

Figure 5.50

If you draw a line down the middle of a butterfly, the two halves match. When this happens, the line is called a **line of symmetry**.

Figures that match exactly when folded in half have a **line of symmetry**. The figures below have lines of symmetry. Some figures can be folded in more than one way to show symmetry. Each fold line is called a **line of symmetry**.

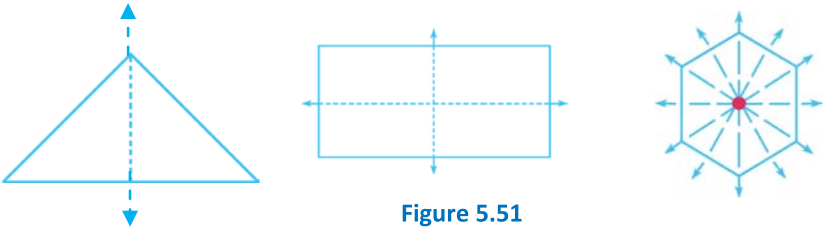
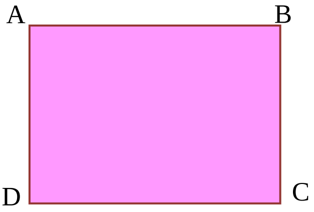


Figure 5.51

Activity 5.6

Material: A square piece of paper.



1. Fold your square so that A touches B. Where is point D? Now try folding it so that A touches D. Where is point C? Try folding it so that B touches C. Try folding it so that C touches D.
2. Open out your square.

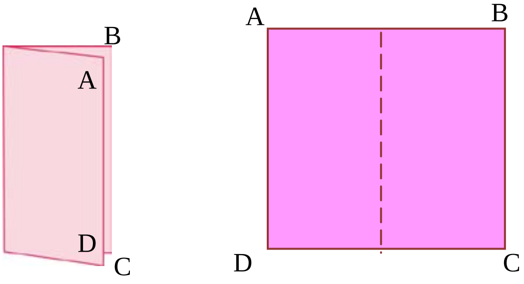


Figure 5.52

The diagram shows the fold line when you put A on top of B. Draw your square in your exercise book.

On your drawing, show the lines that were formed when you folded your square.

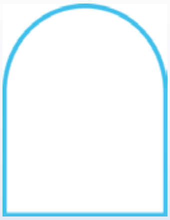
Each time you folded your square, it was divided into exactly two equal parts.

3. Can you find another way to fold your square in to exactly equal parts? Which corner will you match up with A this time? Try it again. Which corner will you match up with B?
4. Add any more fold lines that you have found to your diagram.
5. Draw an equilateral triangle on a square paper. Cut it out. How many lines of symmetry can you find in an equilateral triangle?

Example 6

Determine which figures have line symmetry. Draw all lines of symmetry.

1.



2.

3.

Solution

1. Symmetry

2. Symmetry

3. No Symmetry

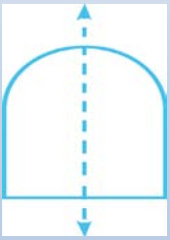
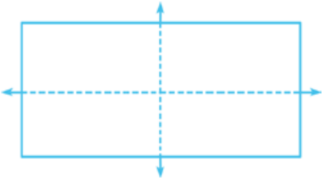


Figure 5.53

Exercise 5.H

1. Identify weather each of the following statements is True or False.

a)

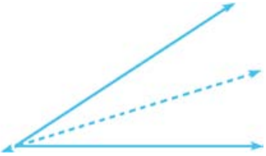


A rectangle has two lines of symmetry.

c)

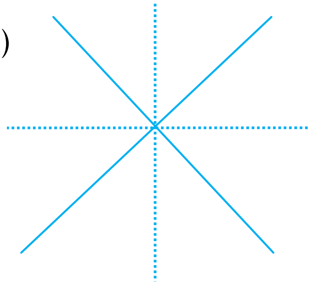
The rattail comb shown has no lines of symmetry.

b)



Any angle has one line of symmetry

d)



The letter X has two lines of symmetry.

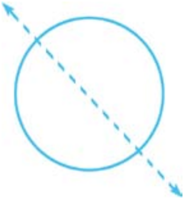
e)

The letter F has a line of symmetry.

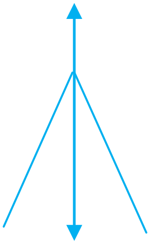
Figure 5.54

2. Tell whether the dashed line is a line of symmetry. Write yes or no.

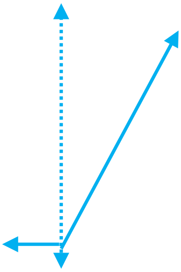
a)



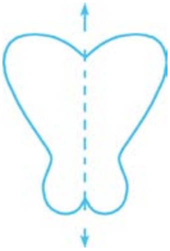
c)



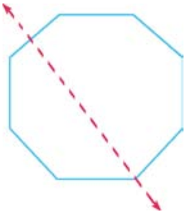
e)



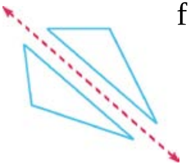
g)



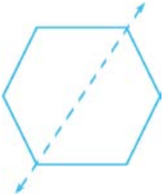
b)



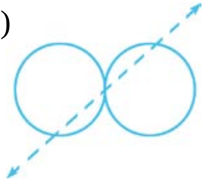
d)



f)



h)



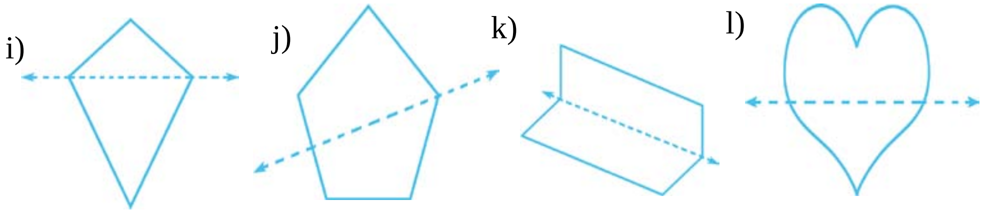


Figure 5.55

3. Trace each figure. Draw all lines of symmetry.

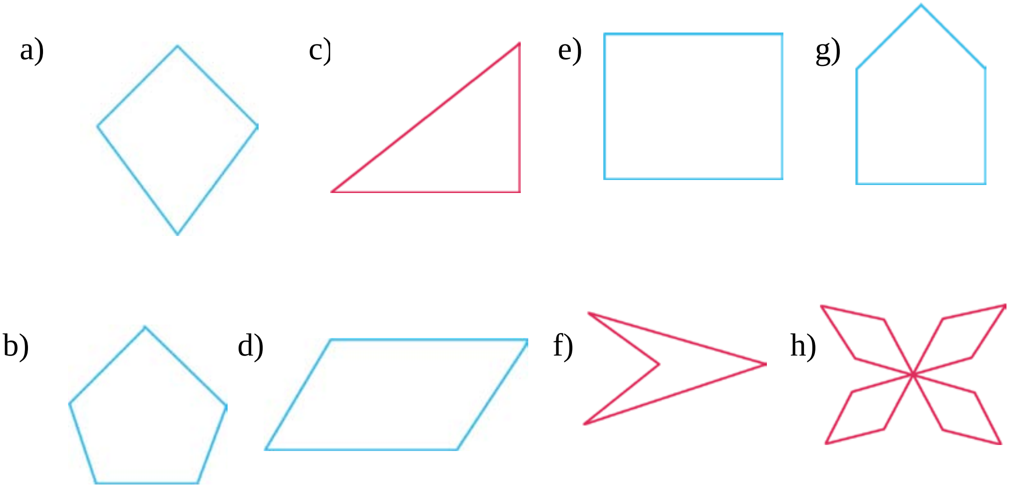


Figure 5.56

4. How many lines of symmetry does
   a) an isosceles triangle have?
   b) an equilateral triangle have?
5. How many lines of symmetry can you find in the picture below?

Figure 5.57

6. Fold a piece of paper in half. Cut out a figure on the fold. Is the cutout symmetric? Where is the line of symmetry?

7. How many lines of symmetry can you find in a circle?

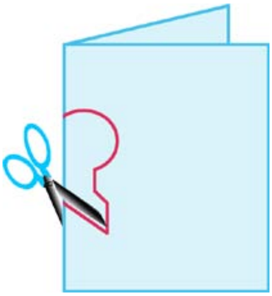


Figure 5.58

5.5. Measurement

Here you will learn about the perimeter and area of rectangles and squares, and solids in everyday life like cubes, cuboids, cylinders, cones and spheres.

5.5.1. The Perimeters and Areas of Squares and Rectangles

Ato Negash wants to put a fence around a section of his back yard so his dog can play. How much fencing will he need?

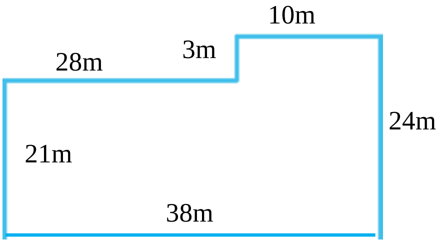


Figure 5.59

Ato Negash needs to know the perimeter of the section he wants to fence so he can know how much fencing material to buy. The **perimeter** (p) of any closed figure is the distance around the figure. You can find the perimeter by adding the measures of the sides of the figure.

P= 28+3+10+24+38+21

P= 124m

The perimeter of Ato Negash's dog run is 124 meter. So, Ato Negash needs 124 meter of fencing.

There is an easier way to find the perimeter of a rectangle. Since opposite sides of a rectangle have the same length, you can multiply the **length** by 2 and the **width** by 2. Then add the products.

Perimeter of a Rectangle

The **perimeter** of a rectangle is two times the length (ℓ) plus two times the width (w). That is,

$P= 2\ell+2w$

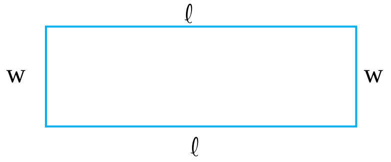


Figure 5.60

Example 7

Ayantu has a rectangular vegetable garden in her back yard that is 6.3 meters long and 2.8 meters wide. She wants to put a border around her garden to keep the rabbits out. How much border will she need?

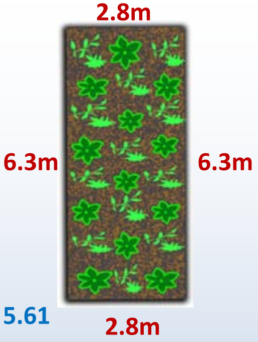


Figure 5.61

Solution: $P= 2\ell+2w$

$P= 2\times6.3+2\times2.8$....... Replace ℓ with 6.3 and w with 2.8.

$P= 12.6+5.6$

$P= 18.2$

The perimeter of Ayant'us garden is 18.2 meters. So, Aynatu needs 18.2 meters of boarder.

An easy way to find the perimeter of a square is to multiply the length of one side by 4. You can use this formula because each side of a square has the same length.

Perimeter = 4x length of one side

or perimeter of a square

$p = 4s$

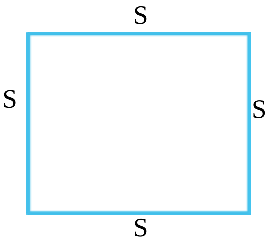


Figure 5.62

Example 8

Find the perimeter of a square whose sides measure 23.4 cm.

Solution:

$P = 4S$

$P = 4 \times 23.4$ ......Replace S with 23.4

$P = 93.6$ cm

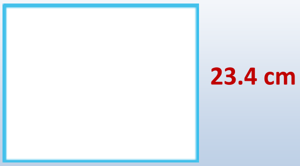


Figure 5.63

Activity 5.7

I. Area of a square

- On square paper, draw a 5cm by 5cm square as shown at the right.
- The area of a geometric figure is the number of square units needed to cover the surface within the figure.

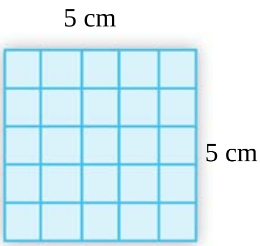


Figure 5.64

Discuss

1. How many squares are found within this square?
2. How does the area relate to the length of the square?

II. Area of a rectangle

- On square paper, draw a rectangle with a length of 6 units and a width of 4 units as shown at the right.

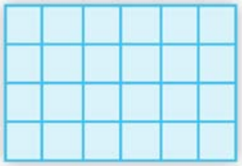


Figure 5.65

Discuss

1. How many squares are found within this rectangle?
2. How does the area relate to the length and width of the rectangle?

Suppose a sport committee decides to build a volley ball playing field that has the measurements at the right. What is the area of the field?

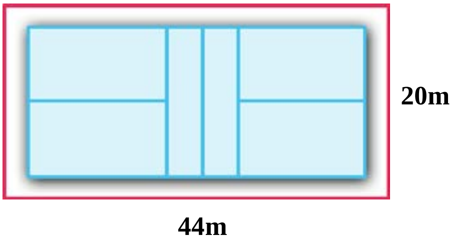


Figure 5.66

Before we can answer this question, we need to understand the concept of area. Area is the number of square units needed to cover a surface.
The rectangle at the right has an area of 24 square units.

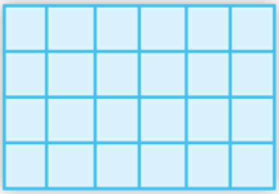


Figure 5.67

Some units of area are the square kilometer (km ^{2}), square meter (m^2), square centimeter (cm^2) and square millimeter (mm^2). Another way to find the area of a rectangle is to multiply.

Area of a Rectangle

The area of a rectangle is the product of its length (ℓ) and width (w). That is , A= ℓ.w

Now we can find the area of the volleyball playing field.

A = ℓ.w

A = 44×20………Replace ℓ with 44 and w with 20.

A = 880

The Area is 880 square meter.

Example 9

Find the area of a rectangle with a length of 12cm and a width of 5cm.

Solution

A= ℓ.w

A=12 × 5………Replace ℓ with 12

A=60

The area of the rectangle 60 square cm. You may check by counting squares.

Can you find area of a rectangle which is 20 cm by 4cm?

Example 10

Semira wants to cover her strawberry garden with nylon net to keep the birds from eating the strawberries. The garden is 12.5 meter long and 7.25 meter wide. How much net does she need to cover her garden?

Solution

A = ℓ × w

A = 12.5 × 7.25……… Replace ℓ with 12.5 and w with 7.5

A = 90.625

Semira needs 90.625 square meter of nylon net.

Since each side of a square has the same length, you can square the measure of one of its sides to find its area.

Area of a Square

The area of a square is the square of the length of one of its sides.

That is, $A=S^2$

Example 11

Find the area of the square at the right.

Solution:

$A = S^2$

$A = (1.5)^2$ …………Replace S with 1.5.

The area of the square is 2.25 m^2

Figure 5.68

The following group work will help you to see the relationship between areas and perimeter.

Group work 5.7

Work in groups

Materials: square paper, scissors.

- Draw a rectangular shape with a perimeter of 48, staying on the lines. Examples of rectangular shapes with a perimeter of 14 centimeters are presented below

Figure 5.69

- Cut out your rectangular shape. Find the area by counting the number of squares. Compare with other members of the group.

Discuss

- Describe the perimeter of each rectangular shape.
- Describe the area of each rectangular shape.
- What can you conclude about the relationship between area and perimeter?

Now, let's solve a simpler problem:

Chuchu wants to carpet her L-shaped dining room and living room area. How much carpet will she need?

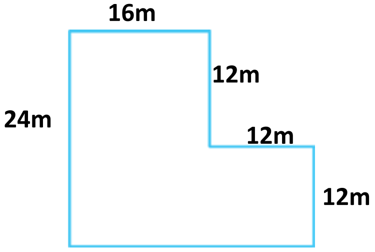


Figure 5.70

What do you know in this problem? You know the dimensions of each room by looking at the diagram. What do you need to find? You need to find the area of the dinning room and living room.

To find the area of the L-shaped rooms, you can first solve a simpler problem. Divide the L-shape in to two regions. Find the area of each region. Then add the area of the regions together to find the total area.

Find the area of region X.

$A= \ell \times w$

$A= 16 \times 12$

$A= 192m^2$

Find the area of region Y.

$A = \ell \times w$

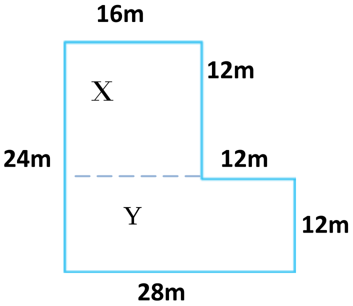


Figure 5.71

A = 28×12

A = 336m²

Add to find the total area.

$192m^2 + 336m^2 = 528m^2$

Chuchu will need $528m^2$ of carpet.

Check your solution by solving the problem another way.

Divide the L-shape area differently and find the area.

Exercise 5.I

1. Find the perimeter and area of each figure.

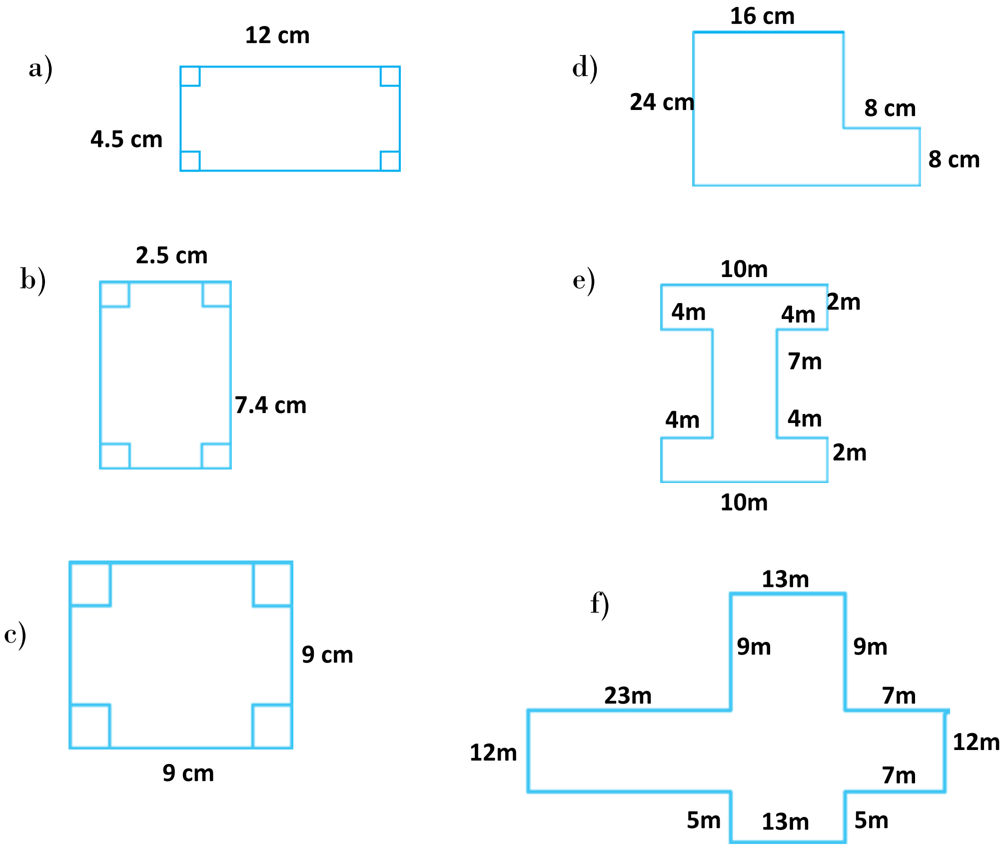


Figure 5.72

2. A rectangular ground is 200 m long and 85 m wide. A cyclist goes around it 6 times. What distance does he cover?
3. A rectangular flower garden 12m long and 9 m wide is divided in to equal sections for 6 kinds of flowers. What are three possible perimeters for one section of the garden.
4. How many 1-meter square tiles are needed to cover the floor of a kitchen that is 16m by 10m?
5. International soccer fields are rectangular and measure 100 meters by 73 meters. A new soccer field needs to be covered with sod. How many square meters of sod will be needed for the field?
6. A cement walk 2.5 m wide surround a pool that is 12m by 25m. What is the area of the walk?

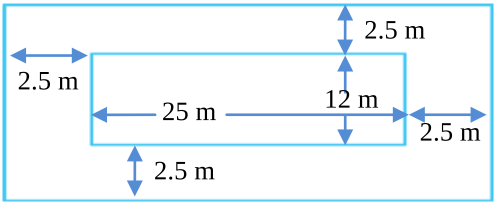


Figure 5.73

5.5.2 Nets of Cubes and Rectangular Prisms

Remember that a flat or plane shape, such as a square, rectangle or triangle, has length and width. It has **two dimensions**. What can we say about a box? All of its faces are rectangles, so it has plane faces. However, as well as length and width, a box has height. It has **three dimensions**.

A **prism** is a three-dimensional shape, which means it has length, width and height.

A prism also has another special property. If a prism is cut at any point along its length, so that the cut is perpendicular to its length, the plane face formed will always be the same shape and size. The face exposed by such a cut is called the **cross-section** of the prism.

The following shapes are all prisms.

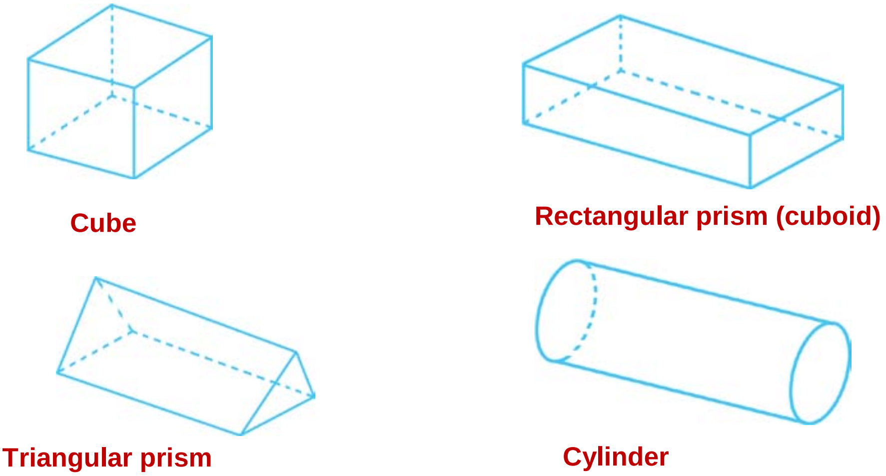
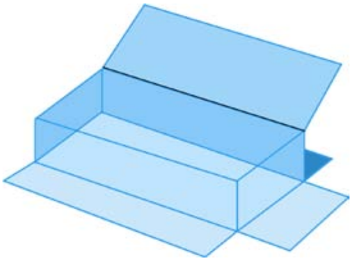


Figure 5.74

Nets



If you remove the surface from a three-dimensional figure and lay it out flat, the pattern you make is called a **net**.

Nets allow you to see all the surfaces of a solid at one time. You can use nets to help you find the surface area of a three-dimensional figure.

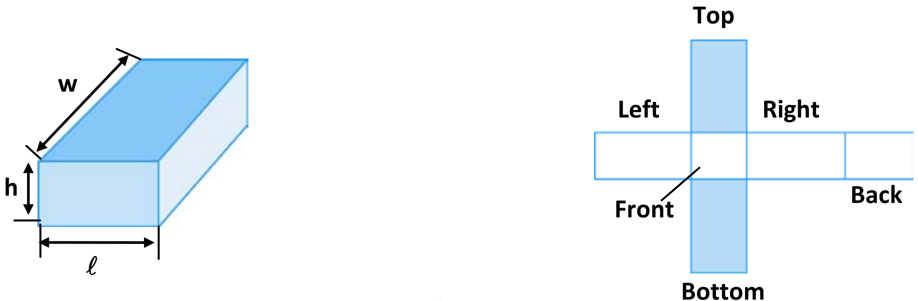


Figure 5.75

Group work 5.8

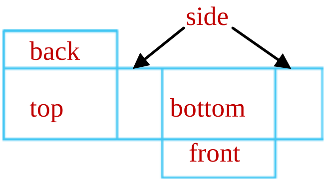
Work in groups

Materials: A box that is 10cm by 4cm by 5cm, graph paper, pencil.

Unfold or cut apart the box.

It should resemble the frame at the right.

- Trace each side of the box on to your graph paper to make a figure like the one at the right.
- Label the dimensions of each rectangle on the graph paper.



Discuss

What is the area of each base and the other four faces? To help you, copy and complete the following chart.

	Dimensions	Area
Front		
Back		
Top		
Bottom		
Left side		
Right side		

Example 12

Make a net, or a pattern for the rectangular prism which is 8cm by 3cm by 5cm.

Solution.

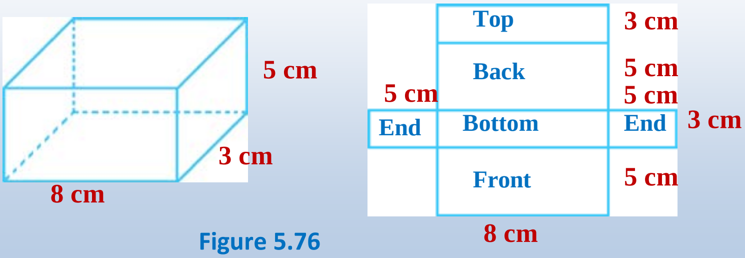


Figure 5.76

Check. Trace the net on the right and cut it out. Join it up to make a cuboid.

Exercise 5.J

1. Make nets for each of the following cubes.

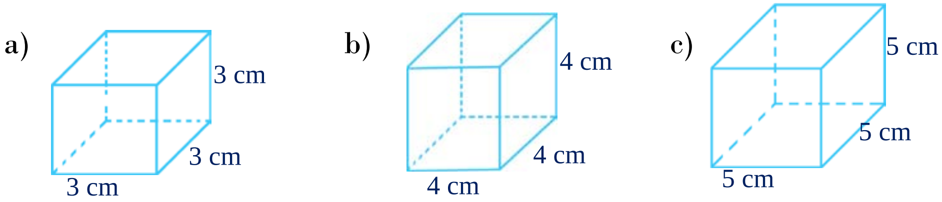


Figure 5.77

2. Make a cube from a net using one of these two methods.

Method 1:
Attach 6 identical squares together like this. Fold and tape to make a cube. Leave the lid of the box open.

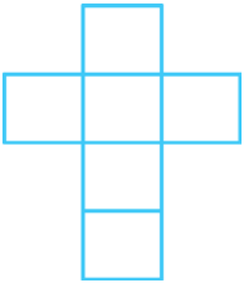


Figure 5.78

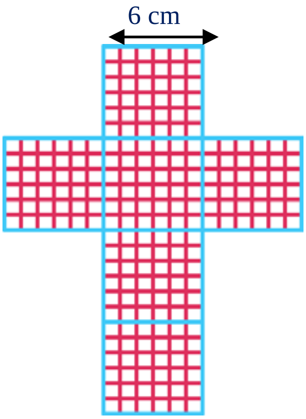


Figure 5.79

Method 2: Draw a net like this on a sheet of centimeter squared paper.

- Stick it on to card.
- Cut, fold and tape to make a cube, leaving the lid open.

3. Maritu made a cube with each edge 3cm long.
   a) Draw a net for her cube on centimeter squared paper.
   b) Check that it makes a cube.

4. Draw a net like this on a sheet of centimeter squared paper.
    - Stick it on to card.
    - Cut, fold and tape to make a cuboid, leaving the lid open.
    - Decorate the faces.

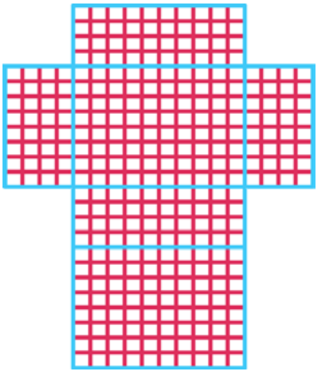


Figure 5.80

5. Lemma used

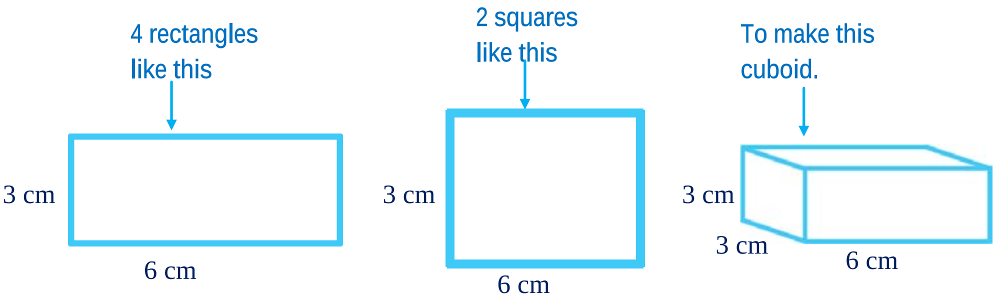


Figure 5.81

a) Draw a net for this cuboid on centimeter squared paper.
b) Check the net by folding.

5.5.3. The Volumes of Cubes and Rectangular Prisms

We can refer to the idea of size to solid figures (three dimensional figures). For example, when we compare the sizes of two boxes we decide which box has more space inside it. The size of a solid figure is called its **volume**.

In order to find the volume of a solid figure, we compare it with another solid figure, usually a smaller one. Then we attempt to fill the given solid figure with unit space figures and count how many are required to fill it. Observations can lead us to the conclusion that a cube is the best unit to use in measuring volumes of solid figures.

Solids such as cubes and cuboids have **faces**, **vertices** and **edges**.

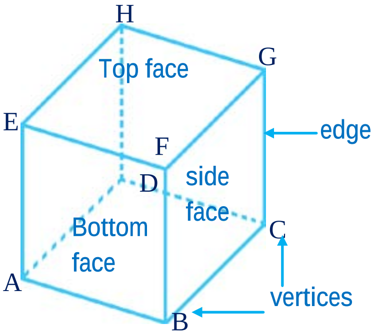


Figure 5.82

Study this cube:

when we examine the cube, we find that it has:

- 6 faces-ABCD is the bottom face
- 12 edges –AB, BF are edges
- 8 vertices –A, B, C, D are vertices.

We can also see that:

- **opposite sides and faces are parallel.**
- **adjacent faces are perpendicular to each other.**
- **adjacent faces meet in an edge.**

For convenience, we can use letters to identify and name the faces, vertices and edges.

In the above cube:

- **ABCD is the bottom face which is equal to the top face EFGH.**
- **ABFE is the front face which is equal to the back face DCGH.**
- **BCGF is the side face which is equal to the side face ADHE.**

Notice that we measure a figure with a unit of the same kind as the figure being measured. (we measure a segment with a unit segment, a plane region with a unit square region). A solid figure is measured with a **cubic unit** (a solid figure in the shape of a cube).

The standard unit of volume used in the metric system is the **cubic meter**. A cubic meter is a cube with each of its edges 1 meter long. Another standard unit of volume is a **cubic centimeter**, which is a cube with all its edges 1 centimeter long.

In the figure below the size of the box ABCDEFGH is measured by the size of the unit cube shown.

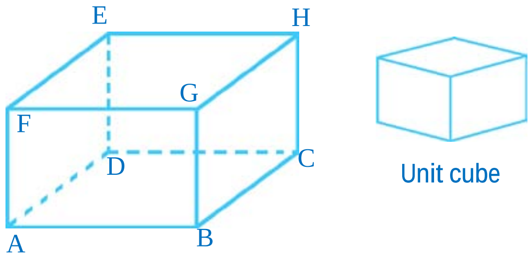


Figure 5.83

We see that about 24 unit cubes are needed to fill the box. Thus the volume of the box is 24 cubic units. Observe that the bottom layer has 3 rows cubes with 2 in each row and there are 4 such layers.

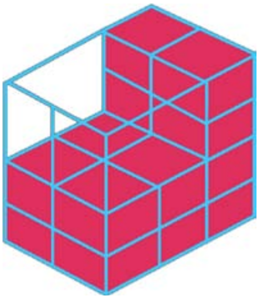
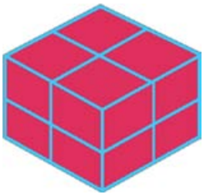


Figure 5.84

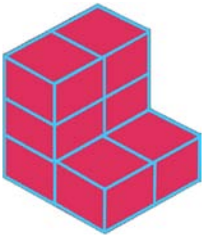
Activity 5.8

Count the number of unit cubes to find the volume of each of the following solids.

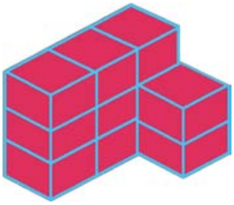
a)



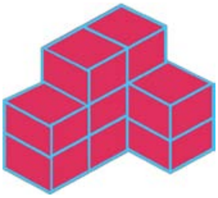
b)



c)



d)



e)

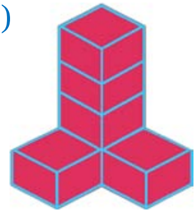


Figure 5.85

Group Work 5.9

Hana has decided to store her magazines in boxes inside the trunk. She wants to keep each subscription together. She has collected boxes that will stack neatly, and completely fill her trunk.

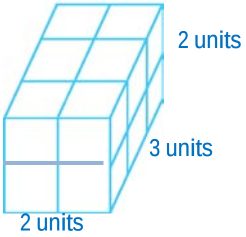


Figure 5.86

Work with a partner.

Materials: a medium-sized box and sugar cubes.

- Estimate how many cubes will stack neatly, and completely fill the box.
- Fill the box with cubes.

Discuss

a. Compare your estimate with the actual number of cubes.

b. Suppose you do not have enough cubes to fill the entire box. However, you have enough to cover the bottom of the box with one layer of cubes, with some cubes left over. Can you determine how many cubes are needed to fill the entire box? Describe your method.

c. Suppose you do not even have enough cubes to cover the bottom of the box. Describe a method to determine how many cubes are needed to fill the box.

From the figure above, you can see that 2 × 3 × 2 or a total of 12 boxes will fit inside Hana's trunk. This is the volume of the trunk.

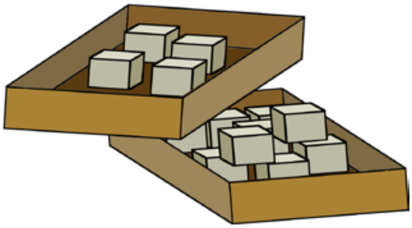


Figure 5.87

Example 13

A shopkeeper arranges boxes of matches as shown the right. How many boxes of matches are there?

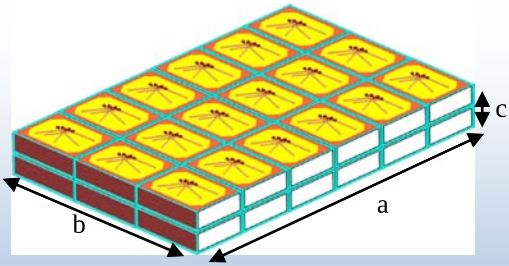


Figure 5.88

Solution: **Observe that there are 6 match boxes alongside a, 3 boxes of matches alongside b, and 2 boxes of matches alongside c. That is, each of the layers has 6 × 3 boxes of matches and there are two layers (bottom layer and upper layer). Therefore, there are 6 × 3 × 2 or 36 boxes of matches.**

Notice that you may also use counting to check whether there are 36 boxes of matches.

Exercise 5.K

1. How many unit cubes are needed to fill the boxes shown below?

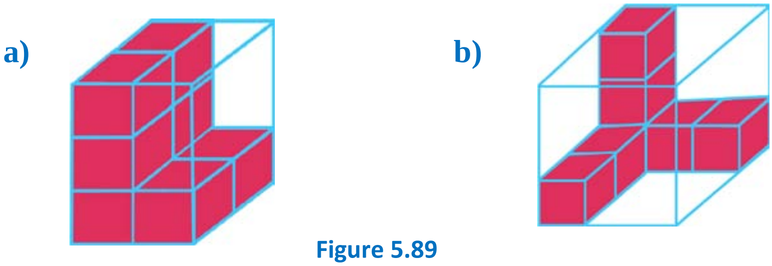


Figure 5.89

2. Count the number of unit cubes to find the volume of each of the following boxes.

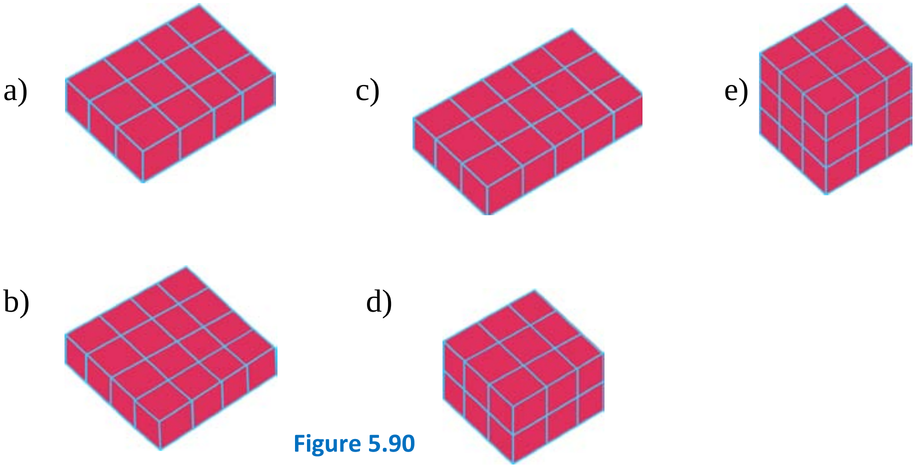


Figure 5.90

3. The Maths club in a certain school invited parents to visit the class and participate in an activity with their children. Parents and students build a prism (sugar cubes) that is shown at the right. What is the volume of the prism built of sugar cubes?

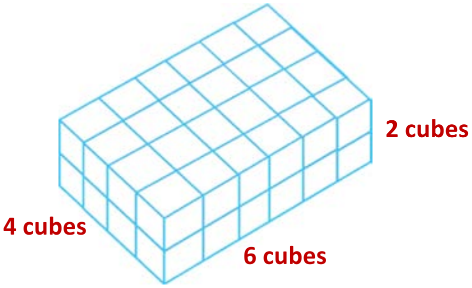
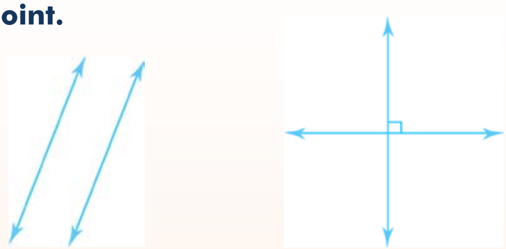


Figure 5.91

UNIT SUMMARY

- **Lines in a plane that never meet are called parallel lines. Lines that intersect to form a right angle are called perpendicular lines. Intersecting lines have exactly one common point.**



Parallel lines Perpendicular lines

Figure 5.92

- **You can use a ruler and set square to draw parallel line to a given line through another point not on the given line.**

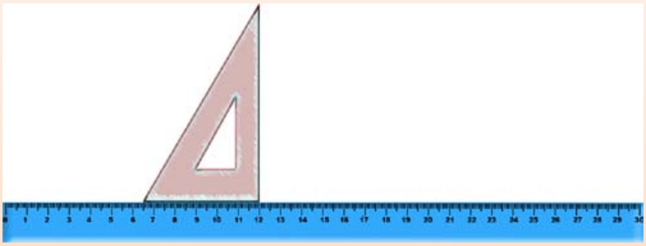


Figure 5.93

- **Keep your ruler very still, and place your setsquare along the ruler, with one of its perpendicular sides next to the ruler and along the line you have drawn.**
- **You can use a ruler and compass to bisect a segment.**

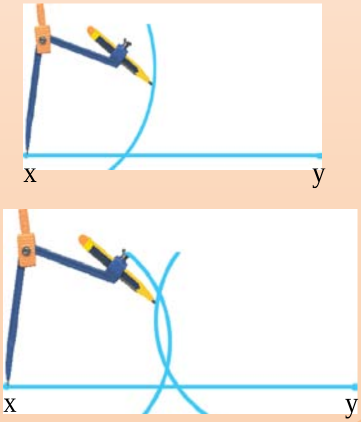


Figure 5.94

- **You can construct a line perpendicular to a given line through a given point.**

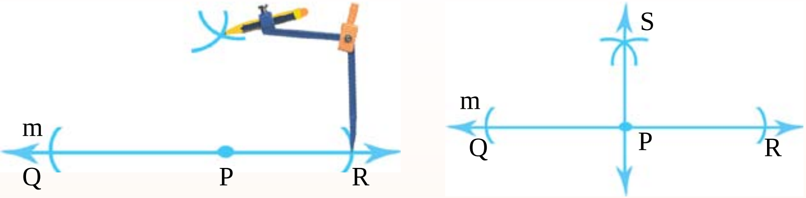


Figure 5.95

- **You can use a protractor to find the measure of an angle.**

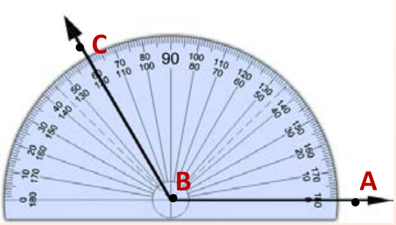


Figure 5.96

- **You can bisect an angle using paper folding.**

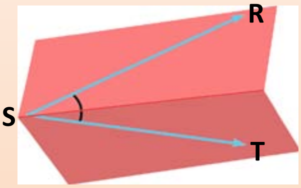


Figure 5.97

- **You can use a ruler and compass to draw an angle and bisect it.**

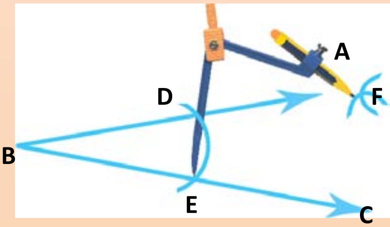
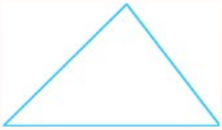
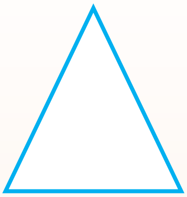
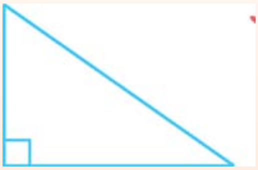
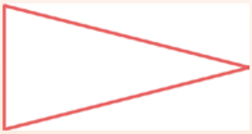
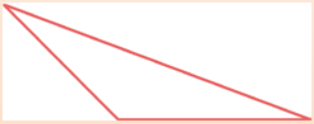


Figure 5.98

Classifying Triangles by Angles	Classifying Triangles by sides
Acute triangle: Three acute angles. 	**Equilateral triangle:** Three congruent sides. 
Right triangle: One right angle. 	**Isosceles triangle:** At least two congruent sides. 
Obtuse triangle: One obtuse angle. 	**Scalene triangle:** No congruent sides.  Figure 5.99

- **Figures that match exactly when folded in half have a line of symmetry.**

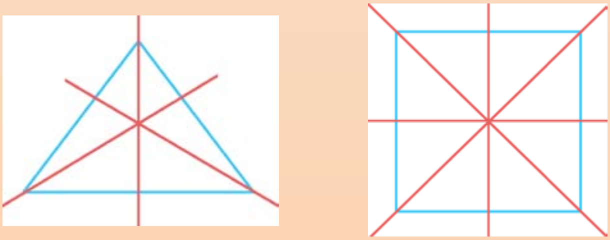


Figure 5.100

- The **perimeter of a rectangle** is two times the length (ℓ) plus two times the width (w).

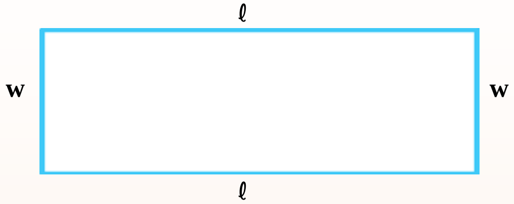


Figure 5.101

- The **perimeter of a square** is given by P= 4S, where S is the length of one side.

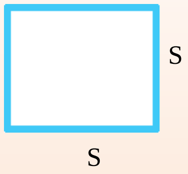


Figure 5.102

- The **area of a rectangle** is the product of its length (ℓ) and width (w), A= ℓ.w
- The **area of a square** is the square of the length of one of its sides (S). A =S^2
- **A cube** is the best unit in measuring volumes of solid figures.

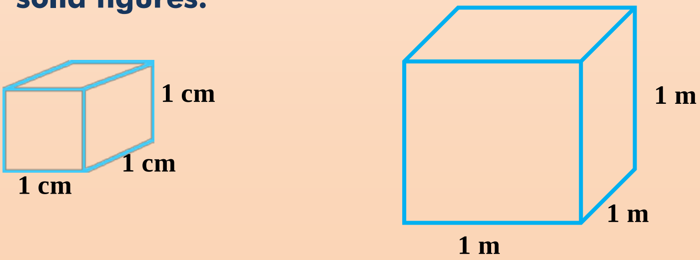


Figure 5.103

REVIEW EXERCISE

1. Tell whether each angle is acute, right, obtuse, straight, or reflex.

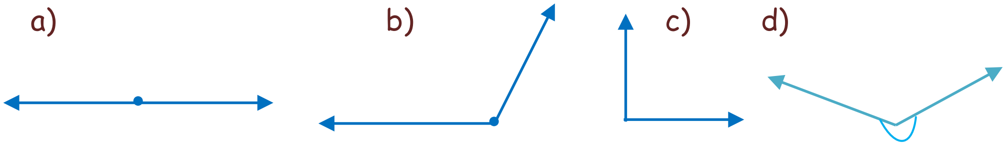


Figure 5.104

2. Which of the following best describes the triangle?

a) scalene, right
b) isosceles, obtuse
c) isosceles, acute
d) scalene, acute

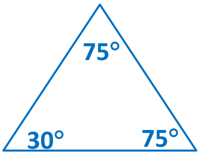


Figure 5.105

3. Look at the clock hands and find the angle

a) between the hands of 5 o'clock.

Figure 5.106

b) between the hands at 8 o'clock.

Figure 5.107

4. Tell whether the lines appear parallel or perpendicular.

a) $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$
b) $\overleftrightarrow{BD}$ and $\overleftrightarrow{AC}$
c) $\overleftrightarrow{EF}$ and $\overleftrightarrow{FH}$
d) $\overleftrightarrow{BF}$ and $\overleftrightarrow{AB}$

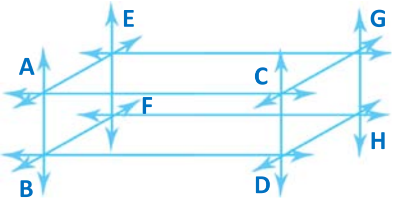


Figure 5.108

5. Based on the angles measures given, which triangle is not acute?

a) 60°, 66°, 54°

b) 90°, 45°, 45°

c) 54°, 54°, 72°

d) 75°, 45°, 60°

6. Decide whether each figure has line symmetry .Check that all the lines of symmetry are drawn.

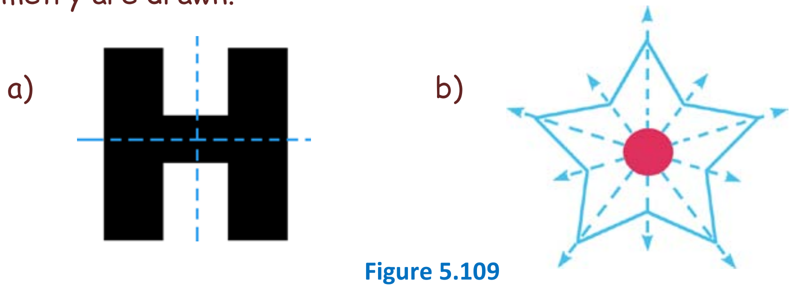


Figure 5.109

7. Find the area and perimeter.

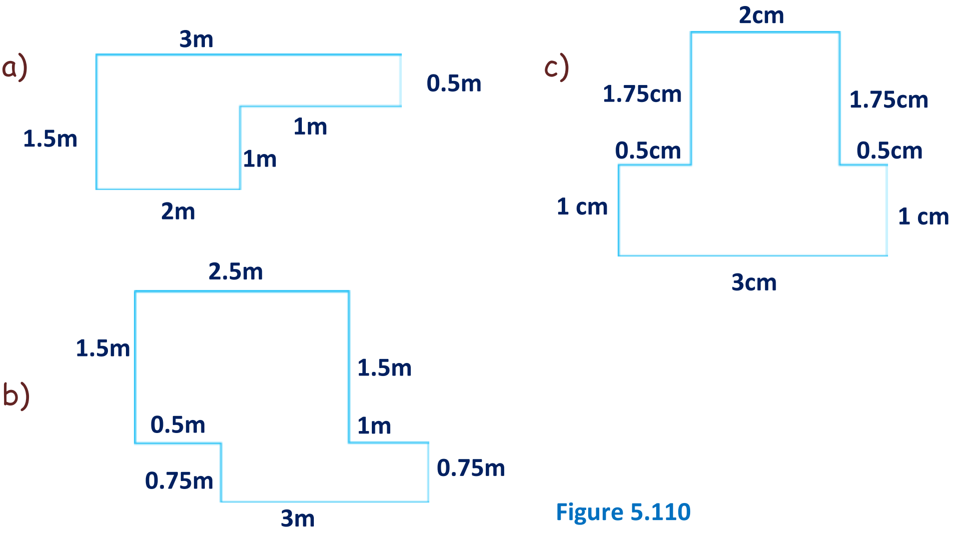


Figure 5.110

8. Find how many cubes each prism holds. Then give the prism's volume.

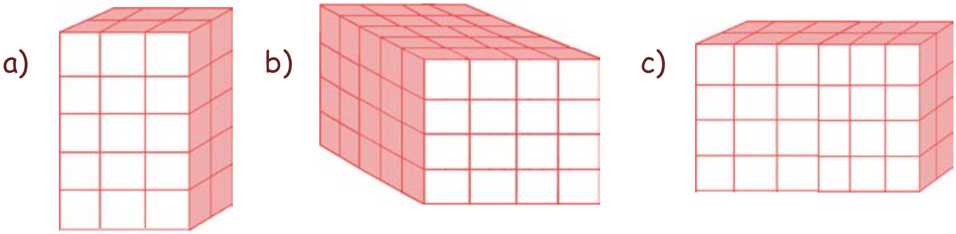


Figure 5.111